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Kindergarten grammars: designing with Froebel's building gifts

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Abstract. A *constructive* approach to the definition of languages of designs is outlined. Froebel's building gifts are used to illustrate the approach.

The kindergarten method

The *kindergarten method* of Frederick Froebel (Kraus-Boelte and Kraus, 1876-1880) is well known to architects and designers (see, for example, MacCormac, 1974) because of its formative influence on Frank Lloyd Wright:

> "The poet's message at heart, I wanted to go to work for the great moderns, Adler and Sullivan; and finally I went, warned by the prophecy and equipped, in fact armed, with the Froebel-kindergarten education I had received as a child from my mother. Early training which happened to be perfectly suited to the T-square and triangle technique now to become a characteristic, natural to the machine-age. Mother's intense interest in the Froebel system was awakened at the Philadelphia Centennial, 1876. In the Frederick Froebel Kindergarten exhibit there, mother found the 'Gifts'. And 'gifts' they were. Along with the gifts was the system, as a basis for design and the elementary geometry behind all natural birth of Form" (Wright, 1957, page 19).

The kindergarten method is based on a series of geometrical *gifts* and a system of *categories* of geometrical forms. In the kindergarten, the child plays with one or another of the gifts to discover its properties and possibilities for design. Whenever he reaches an impasse, he can turn to his mother. She invokes one or more of the categories to suggest a new avenue or direction of play. The child is thus encouraged to think about certain kinds of designs that can be made with the gifts. In this way, he learns about the world, to describe it, and to plan and act in it effectively.

The gifts

> "Here was something for invention to seize, and use to create" (Wright, 1932, page 34).

The gifts are presented to the child in sequence. Individual solids (the coloured balls of gift 1, and the sphere, cylinder, and cube of gift 2) are followed by composite ones (the divided cubes of gifts 3-6). Planes (the *tablets* of gift 7), lines (the *slats*, *sticks*, *rings*, and *string* of gifts 8-12), and points (gift 13) complete the series. Each succeeding gift is spatially more articulate or spatially more abstract.

Gifts 3-6 are used for illustration. These beautiful sets of blocks are called the *building gifts*; they are shown in figure 1.

Gifts 3 and 4 are cubes with edges two inches long. Gift 3 is divided into eight smaller cubes, each having edges one inch long, by three mutually perpendicular cuts. Gift 4 is divided into eight oblong blocks, each two inches long, one inch wide, and half an inch deep, by a single vertical cut and three horizontal ones.

Gifts 5 and 6 are elaborations of gifts 3 and 4, respectively; both are cubes with edges three inches long.

Gift 5 is cut twice in three mutually perpendicular directions to produce twenty-seven equal cubes, each the same as those in gift 3. Twenty-one of these cubes are not divided again; three are cut once diagonally into 'half-cubes'; three are cut twice diagonally into 'quarter-cubes'. Thus there are thirty-nine separate pieces in gift 5.

Gift 6 is produced from twenty-seven oblong blocks, each the same as those in gift 4. Eighteen of these blocks are used as they are; three are cut once lengthwise to make 'pillars'; six are cut once crosswise to make 'squares'. Thus there are thirty-six separate pieces in gift 6.

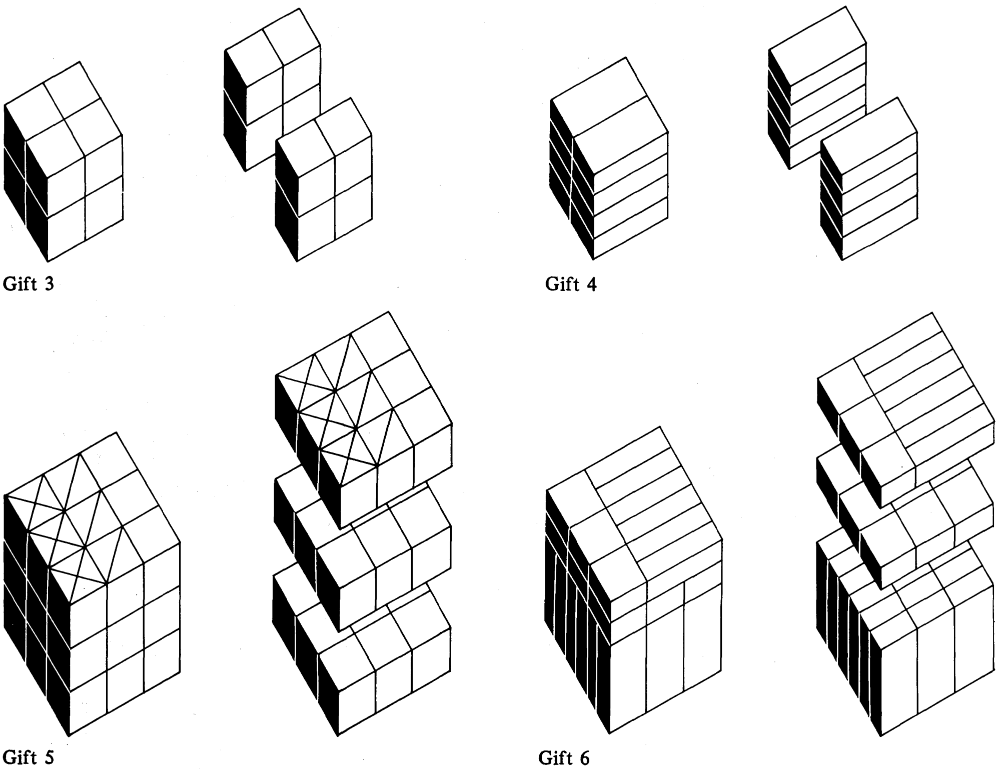


Figure 1. Froebel's building gifts.

The categories

Froebel distinguishes three categories of forms for use with the gifts: forms of *knowledge*, forms of *life*, and forms of *beauty*. Each of these categories is defined by verbal exposition and by example.

Forms of knowledge involve mathematical and logical ideas, such as, number, proportion, equivalence, and order. These ideas serve to define natural divisions of a gift and to suggest ways of rearranging or transforming these parts. Some forms of knowledge for each of the building gifts are shown in figure 2.

Forms of life represent things that can be seen in the outside world, for instance, a group of buildings, a house, a table, a sofa, and so on. Pieces from a gift are arranged in these forms according to relationships observed in other designs. Forms of life made with the building gifts are shown in figure 3.

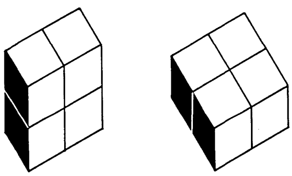
Forms of beauty consist of blocks arranged on a planar surface without stacking to have some kind of symmetry; these patterns may be viewed as ornament. Forms of beauty designed by means of the building gifts are shown in figure 4.

Play

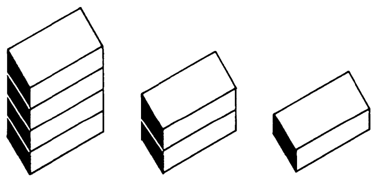
"Each new Gift, after an orderly presentation of it has been made (in order that the child may see it as a whole) is given to him to be used in *free play*, so that he may discover for himself as many of its possibilities as he can. Experience, however, has shown that he soon exhausts his own limited power of observation, and either calls for help, or pushes the blocks to one side as useless things. Before this stage is reached the skillful kindergartner has discerned its approach, and has thrown in some suggestion which adds vividness, or a new idea, to the play" (Harrison and Woodson, 1903, page 80).

The child employs one or another of the gifts in free play; he is expected to use it to design new forms by trial and error. His success in this enterprise depends on his ability to discover alternative ways of putting the pieces of the gift together. His power of observation, like our own in similar circumstances, fails when he can see no further possibilities for arranging elements or when he can see too many. Either way, he cannot think what to do next. In the first case, he cannot see a way to carry his design forward because he cannot see how to add another piece. He needs a way to create new combinations of elements or to deploy known combinations in original ways. In the second case, he cannot proceed because he cannot decide on one way to add another piece to his design from among multiple possibilities. He needs a way to choose one combination of elements from the many or to deploy a given combination in a specific way.

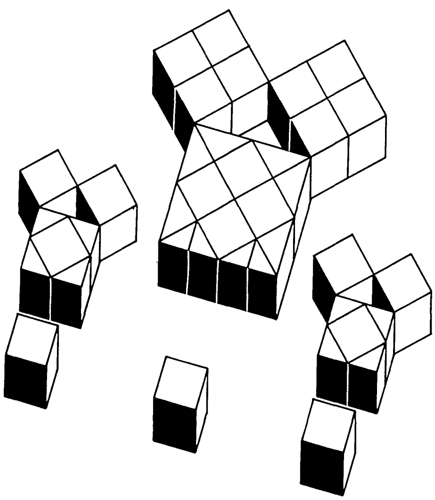
Whenever the child reaches a dead end in his design, he can turn to his mother for a new idea. This is the great advantage of the kindergarten method.



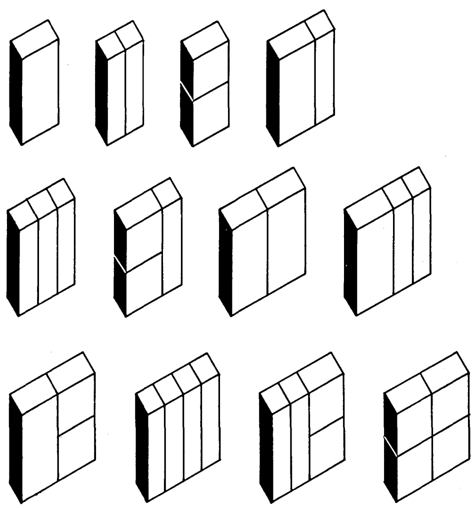
Gift 3: equivalence



Gift 4: division by 2



Gift 5: pythagorean theorem



Gift 6: expanding series

Figure 2. Forms of knowledge made with the building gifts.

Asked for help or anticipating some difficulty, the child's mother invokes one of Froebel's three categories of forms to suggest a new avenue or direction of play. This category or one of its instances helps the child to organize his thought and thereby to increase his power of observation in both the creative and the selective senses described above. For example, thinking about designing a 'house' with the pieces in one of the building gifts suggests some new ways of adding blocks to a design, and at the same time provides some guidance on which combinations of blocks should be employed where. More particularly, a house has a 'roof' which implies a certain arrangement of blocks that must occur in a special place in the design (see, for example, the house in figure 3 made with the pieces of gift 5).

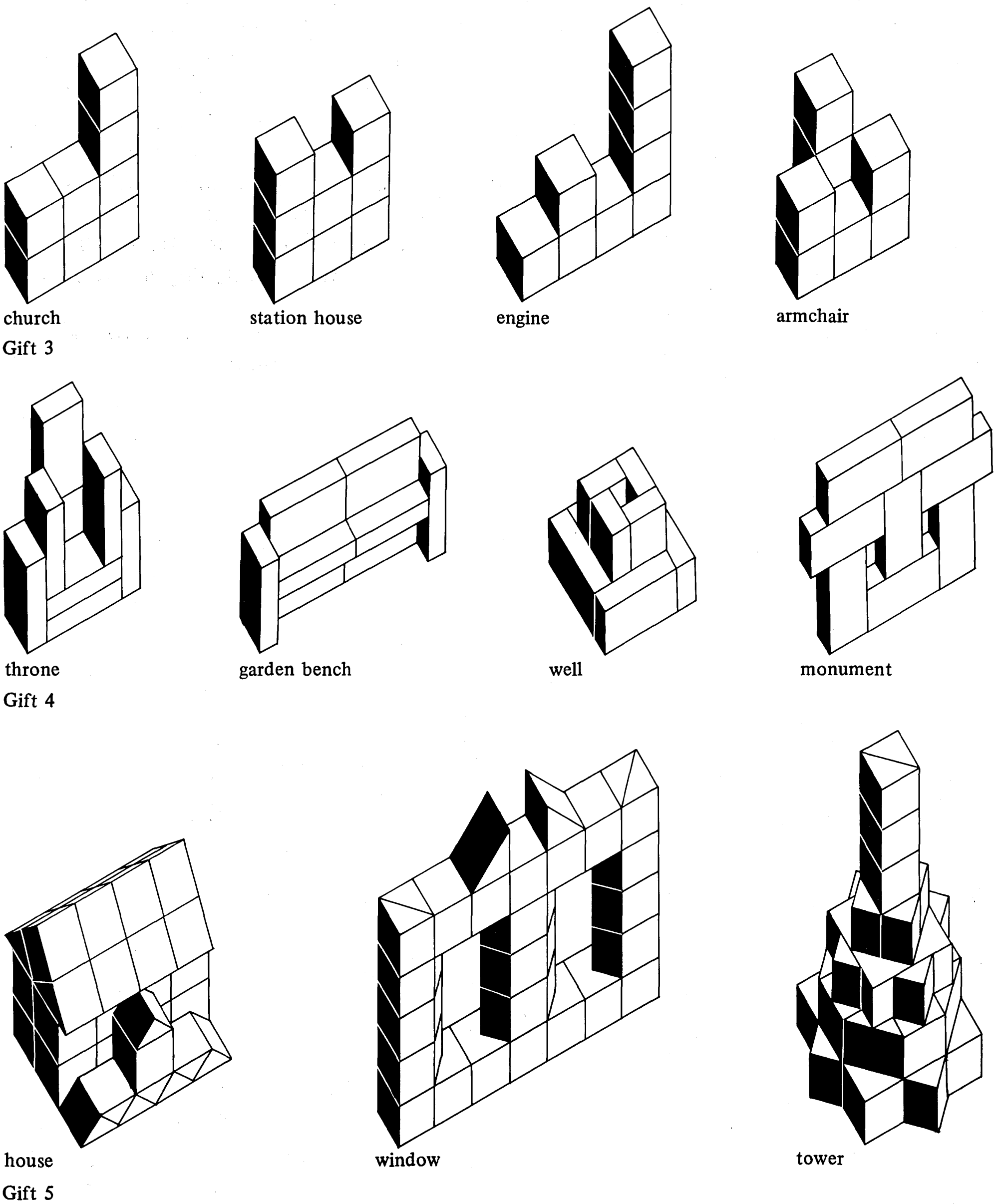


Figure 3. Forms of life made with the building gifts.

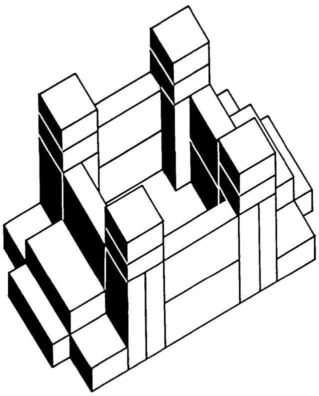
Thus by use of Froebel's categories of form, the child learns to solve his design problems with the gifts and so discovers for himself their properties and possibilities for design.

"The virtue of all this lay in the awakening of the child-mind to rhythmic structure in Nature—giving the child a sense of innate cause-and-effect otherwise far beyond child-comprehension. I soon became susceptible to constructive pattern *evolving in everything I saw*. I learned to 'see' this way and when I did, I did not care to draw casual incidentals of Nature. I wanted to *design*" (Wright, 1957, page 20).

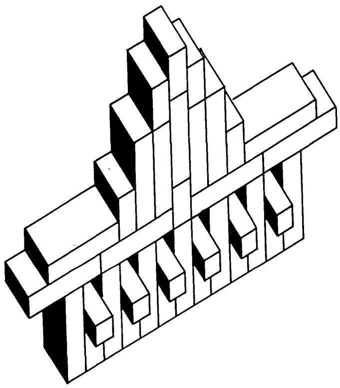
The kindergarten and the studio

It is not surprising that Wright found his kindergarten education so valuable. The gifts are perfectly suited to explore possibilities for spatial design. But more than this, the kindergarten method itself suggests the *studio method*, the most successful educational paradigm used in architecture and design schools today. The very best examples of a young designer and his master in the studio correspond almost exactly to the child and his mother in the kindergarten.

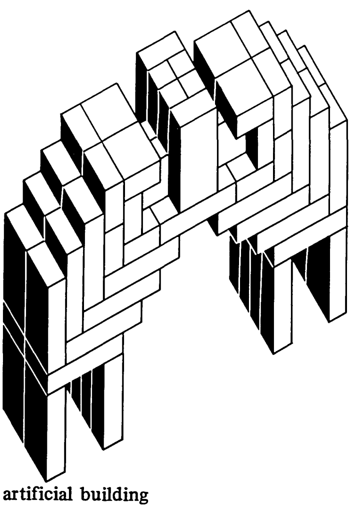
The kindergarten method involves a *vocabulary* of building elements (the distinct pieces for one or another of the gifts) and a system of categories of forms. These categories are invoked to suggest possibilities for design with the elements in



bath



church organ



artificial building

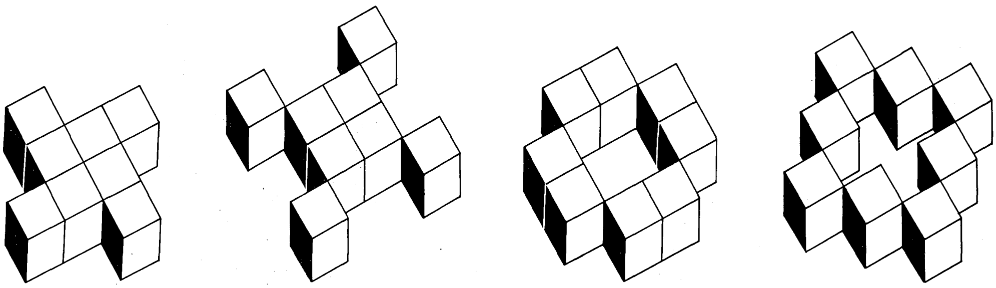
Gift 6

Figure 3 (continued)

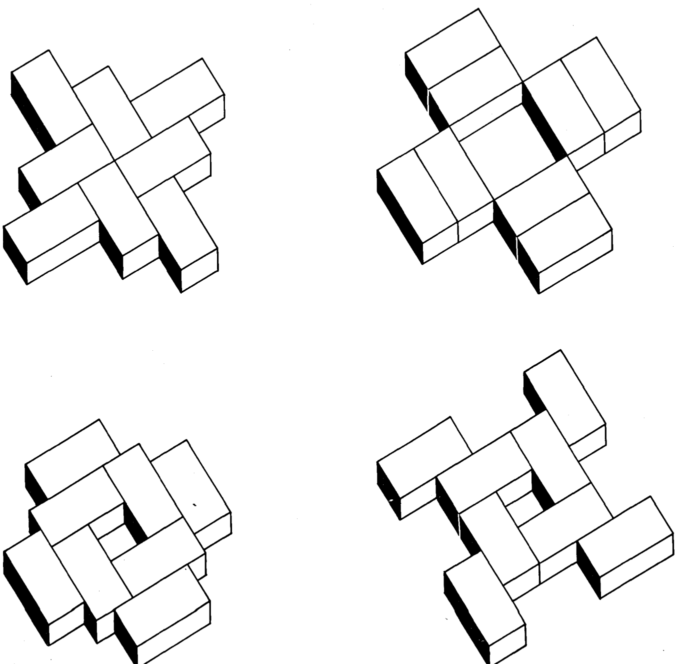
the vocabulary. Each combination of a vocabulary and a system of categories may thus be considered to establish a *language* of designs. Members of this language are designs made up of elements in the vocabulary arranged according to the categories in a more or less intuitive way. Of course, by changing the vocabulary or the system of categories, different languages of designs could be established. The kindergarten method may be viewed as a way to teach such languages.

The studio method, too, aims at teaching languages of designs. But here, a vocabulary may contain certain architectural or structural elements, and the categories may pertain to architectural programmes, building types, historical styles, symbolic references, aesthetic doctrines, and so on.

Following the tradition of the kindergarten, the young designer receives his first vocabulary and system of categories from his master. He learns by trial and error to recognize the designs in the language so established and to create them for himself. Whenever he is lost, he can turn to his master for guidance. The master may invoke an appropriate category or one of its instances to help the young designer understand the language better, or he may, from time to time, suggest another vocabulary and system of categories and thereby establish another language for his charge to investigate.



Gift 3



Gift 4

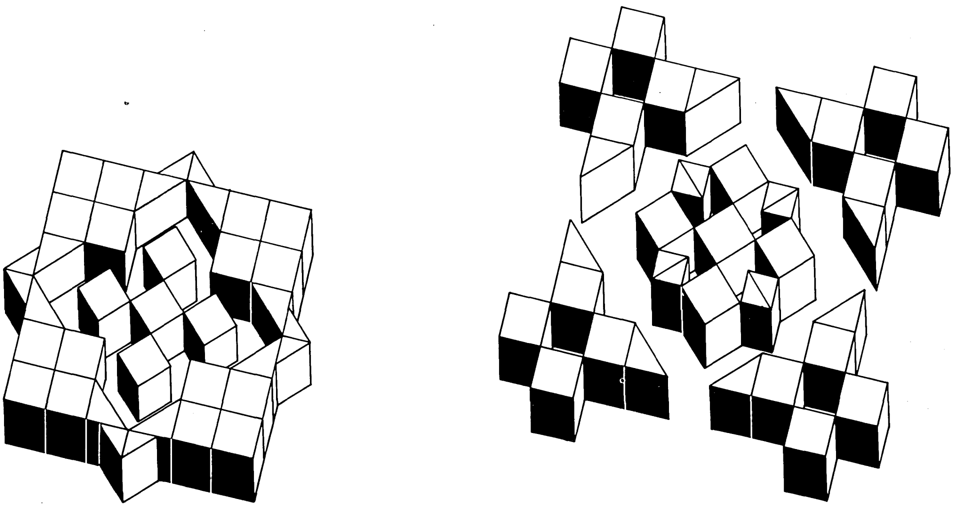
Figure 4. Forms of beauty made with the building gifts.

In this way, the young designer learns to use languages of designs, and sees how to establish new ones.

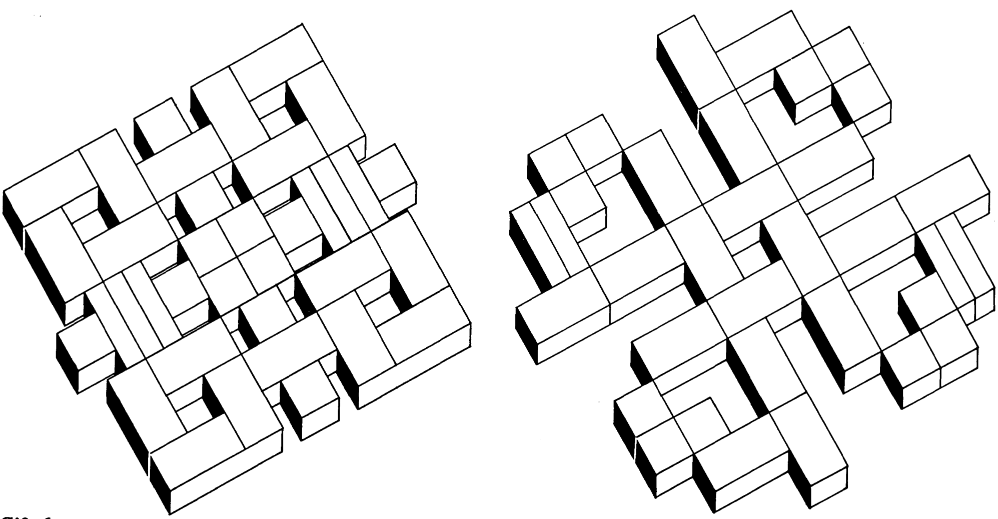
Once the young designer has completed his studio training, he no longer needs his master as a source or interpreter of vocabulary and categories. He can now provide these for himself, and use the languages of designs so established. The lessons of the kindergarten qua studio are thus carried into practice.

Beyond the kindergarten

Languages of designs established in the manner of the kindergarten can be taught in the studio and used in practice. Even so, the kindergarten method is at best merely heuristic. Knowing a vocabulary of building elements and a system of categories to suggest possibilities for design neither determines the extent of a language of designs in any exact sense nor provides the means to construct its individual members in detail. To *define* languages of designs in this way requires an alternative method.



Gift 5



Gift 6

Figure 4 (continued)

In this paper, a *constructive* approach to languages of designs is proposed in the belief that something like it will ultimately replace the kindergarten method both in the studio and in practice.

The proposal develops the idea that a language of designs can be defined from scratch by *rules* which apply to a vocabulary of building elements. Here, rules take the place of categories as construed in the kindergarten or the studio, but differ from them in two important respects. First, where categories can only be invoked to suggest designs in an intuitive way, rules can be applied to construct them in a mechanical (algorithmic) way. Second, where categories usually imply, at least in very general terms, the purpose and meaning of designs, and thus pertain to their 'semantics', rules fix in very specific terms the structure of designs, and thus pertain to their 'syntax'. In this sense, languages of designs defined by the constructive approach may be considered to be interpreted syntactically but not semantically. Once rules have been used to define languages of designs, however, the *aesthetic systems* described by Stiny and Gips (1978a) involving both algorithmically based *constructive* and *evocative* modes of understanding, or perhaps even *model-theoretic* techniques such as those described by Montague (1974, chapters 6 and 7), can be employed to establish the purpose and meaning of designs in these languages. Semantic extensions of languages of designs defined by the constructive approach are expected to be developed in subsequent papers.

The development of the constructive approach focuses on ways to define languages of designs for any given vocabulary of building elements; it extends and refines some ideas about architectural composition originally considered in Stiny (1976). Parts of the discussion rely on the *shape grammar* formalism. The relevant definitions and notation are given in Stiny (1980). Froebel's building gifts are used throughout for purposes of illustration. This return to the devices of the kindergarten is not gratuitous. The pieces of the building gifts may be viewed as simple architectural elements, and thereby provide a rich source of material for architectural composition. Further, these building blocks are not encumbered as are traditional architectural elements, for example, the classical orders, with well-established conventions of usage. As a result, a full range of possibilities for design can be explored without offending deeply engrained but nonetheless conventional architectural sensibilities. Other interesting sets of children's building blocks that can be employed in this way include Froebel's tetrahedral gift (Wheelock, 1913); the Bauhaus blocks designed by Alma Buscher (Wingler, 1969); Lowenfeld's 'poleidoblocs' (Andersen, 1969); and the play blocks described by H G Wells (1911), which are now called Abbatt building bricks.

By showing that the pieces of the building gifts can be combined according to very simple rules to construct designs with greater facility than Froebel's categories allow, it should become evident that constructive methods that can be carried out by a computer are a more dependable and, indeed, more original source of new designs than the intuitive methods of the kindergarten. Using rules instead of intuition, the designer need no longer rely on 'creative inspiration', the 'inventive flash', or 'individual genius'. Once these barriers to clear thinking in design have been removed, we can begin to answer that persistent query: 'Where do designs come from?'

A constructive approach to languages of designs

Languages of designs (shapes) can be defined according to the programme outlined in figure 5. Designs in these languages are constructed by means of shape grammars.

The programme has five stages:

(1) A *vocabulary* of shapes is specified. These shapes provide the basic building elements for design.

(2) *Spatial relations* between shapes in the vocabulary are determined. Building elements are arranged in designs according to these spatial relations.
(3) *Shape rules* are specified in terms of the spatial relations. The structure observed in designs depends on the recurrence of spatial relations used to construct them. The way spatial relations recur is fixed by the shape rules.
(4) Shapes in the vocabulary are combined to form *initial shapes.* Shape rules apply recursively to initial shapes to construct designs.
(5) *Shape grammars* are specified in terms of the shape rules and the initial shapes. Each shape grammar defines a language of designs.

The programme is nondeterministic: multiple choices can be made in each stage. Individual possibilities in one stage may lead to multiple possibilities in a succeeding stage. For example, a single vocabulary can support a variety of spatial relations, and a single spatial relation can supply the basis for a variety of shape rules. Multiplying possibilities from stage to stage in the programme allows for increasingly finer spatial distinctions to be made, and thus provides the constructive machinery to define languages of designs with ever increasing precision and control.

The individual stages in the programme are now elaborated.

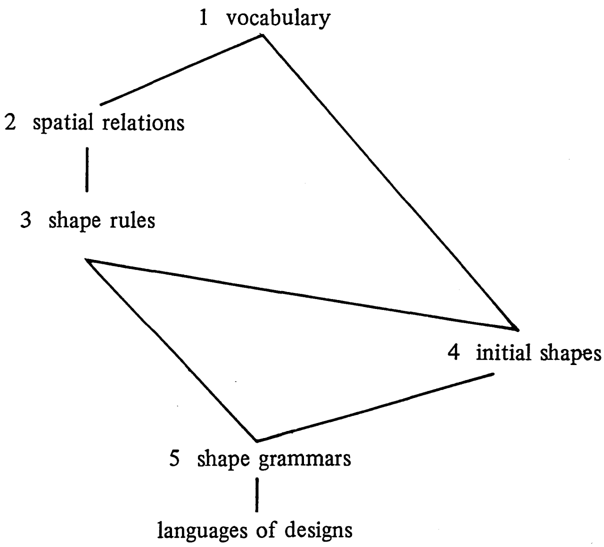


Figure 5. A constructive programme for the definition of languages of designs.

Vocabulary

A vocabulary is a limited set of shapes no two of which are similar. The individual blocks comprising the vocabularies for the building gifts are specified in figure 6.

In general, designs may be constructed by combining euclidean transformations of shapes in a given vocabulary by shape union and shape difference. Unless the use of these operations is restricted, however, the possibilities for design with all vocabularies

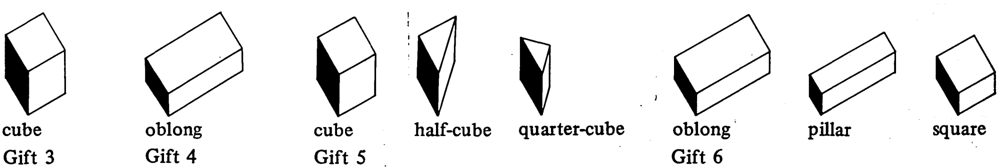


Figure 6. Vocabularies of shapes for the building gifts.

are exactly the same. In the general case, one vocabulary of shapes is as good as the next. The value of a vocabulary in design thus depends on the conventions governing the way its elements are combined. Spatial relations between the shapes in a vocabulary are used to control the manner in which the transformations and shape union and shape difference are used to construct designs.

The equivalence of vocabularies, when the transformations and shape union and shape difference are used without restriction, makes it tempting to consider all design in terms of a standard, canonical vocabulary. This vocabulary when closed under these operations defines the *universe* of all possible designs.

There are two problems with this approach. First, the overwhelming combinatorial complexity of the universe makes it impossible to explore, either by enumeration or by analytical methods, without some preconceived idea of what constitutes an interesting design or type of design. Second, the definition of the universe does not help to frame such ideas; it provides neither useful information about the structure and properties of individual designs nor a basis to partition or order the universe so that interesting designs can be grouped or linked together. To describe the structure and properties of individual designs and thus develop ideas about interesting ones, one must turn to spatial relations.

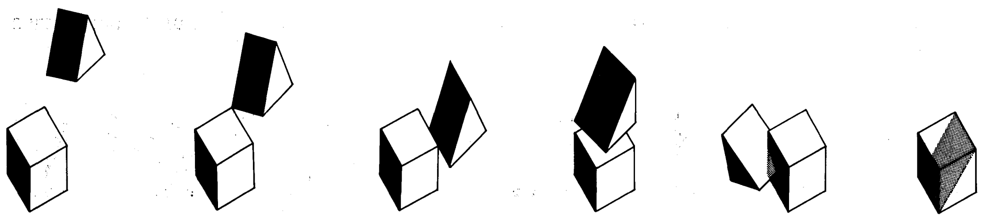
Spatial relations allow for languages of designs to be defined for any given vocabulary. These languages are all proper subsets of the universe; for different vocabularies or different spatial relations, they are usually distinct. The structure and properties of individual designs in each such language depend on the vocabulary and spatial relations on which the language is based.

Spatial relations

Definition

A spatial relation is specified whenever any collection of shapes is considered to form a recognizable arrangement or gestalt. For example, six different spatial relations between the cube and half-cube of gift 5 are shown in figure 7. In the first of these spatial relations, the two volumes do not touch in any way; in the second, they share a vertex; in the third, an edge of one overlaps an edge of the other; in the fourth, a face of one overlaps a face of the other; in the fifth, they interpenetrate; and in the sixth, the half-cube is totally inside the cube. Of course, there is no limit on the number of spatial relations that can be specified for these volumes; each time their dispositions are changed another spatial relation is determined.

A spatial relation may form a perceptual gestalt in the sense that one sometimes finds it difficult not to see a collection of shapes as an organized whole. The smaller building gifts, gifts 3 and 4, may be considered to specify spatial relations of this type. Alternatively, a spatial relation may merely form a conceptual gestalt in the sense that one sometimes treats a collection of shapes as a unified arrangement for purposes of description or explanation. The larger building gifts, gifts 5 and 6, specify spatial relations of this type.



(a) (b) (c) (d) (e) (f)

Figure 7. Some spatial relations between a cube and a half-cube.

The easiest way to specify a spatial relation is to point to an example of it, that is, to distinguish the shapes that have the spatial relation and to show how they are arranged by giving their shape union. The spatial relation of figure 7(a) is specified in this way in figure 8(a). The individual volumes in this spatial relation must be given so that their shape union can be parsed unambiguously. Alone, the shape of figure 7(a) would represent no determinate spatial relation; it could just as easily correspond to the spatial relation between the shapes of figure 8(b) as it does to the spatial relation between the more familiar volumes of figure 8(a). It will be said that any transformation of an arrangement of shapes corresponding to a spatial relation also satisfies the spatial relation. Thus the shapes shown in figure 8(c) satisfy the spatial relation specified in figure 8(a).

More precisely, a spatial relation is specified by a set of shapes. The shapes in a set S' have the spatial relation specified by a set of shapes S whenever there is a bijection $f\colon S \to S'$ and a transformation τ such that for every shape s in S, $f(s) = \tau(s)$. In this case, the set S' contains the same number of shapes as the set S specifying the spatial relation, and every shape in S' is identical to the transformation τ of some shape in S.

If the shapes in two sets have the same spatial relation, then the shape union of all of the shapes in the first set is geometrically similar to the shape union of all of the shapes in the second set. The converse of this statement, however, is not true, as is shown by the spatial relations of figures 8(a) and (b). Each of these spatial relations gives an alternative way to parse similar shapes.

In some cases, it may be more proper to define spatial relations in terms of only some of the transformations or in terms of more general transformations of shapes. For example, if mechanical equilibrium is taken to be a factor in design, then spatial relations would more properly be defined in terms of translation and scale or finite compositions of these. Otherwise, two sets of shapes could have the same spatial relation, but the shape union of all of the shapes in one set could have mechanical equilibrium, and the shape union of all of the shapes in the other set could not. Alternatively, it is sometimes desirable to consider more general kinds of spatial relations. In this case, spatial relations could be defined by sets of parameterized shapes. The shapes in a set S' have the spatial relation specified by a set of parameterized shapes S whenever there is a bijection $f\colon S \to S'$, an assignment g of values to all of the variables associated with the elements of S, and a transformation τ

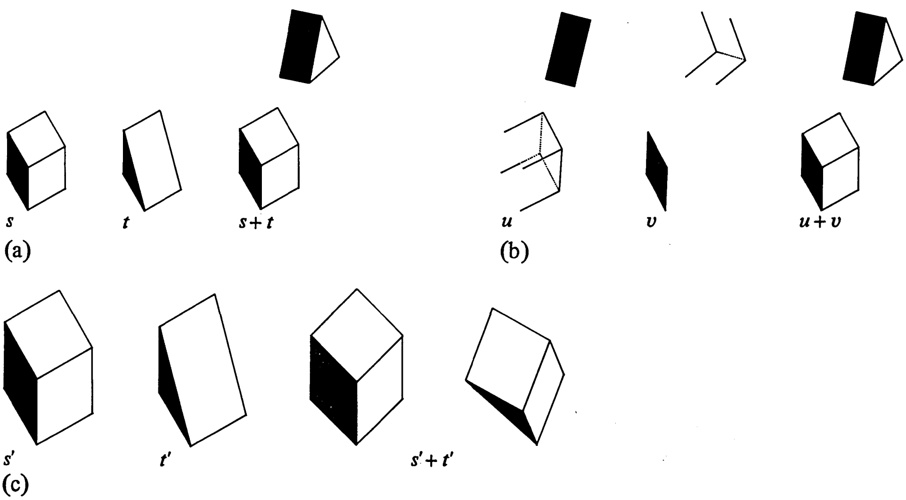


Figure 8. Specifying spatial relations.

such that for every parameterized shape s in S, $f(s) = \tau[g(s)]$. Neither of these alternatives is used in this paper; they are important to keep in mind for other design applications. All of the results developed here can be restricted or generalized in either of these ways.

Spatial relations for the building gifts

In the kindergarten, the following rhyme is recited to suggest ways of combining pairs of blocks in one or another of the building gifts:

> "Face to face put. That is right.
> Edges now are meeting quite.
> Edge to face now we will lay,
> Face to edge will end the play"
> (Harrison and Woodson, 1903, page 106).

The rhyme directs one to combine two pieces of a building gift so that a face of one touches a face of the other and their edges coincide. One can produce such combinations by lining up blocks with one's thumb and forefinger as shown in figure 9 for the oblong and pillar of gift 6.

More precisely, it will be said that all spatial relations determined according to these informal criteria satisfy the following requirements:
(1) Each spatial relation is specified by a pair of blocks in one of the building gifts, that is, by a pair of isometries of shapes in one of the vocabularies of figure 6.
(2) A face of one block overlaps a face of the other block so that these faces share a vertex, and the edges of the respective faces intersecting at this point coincide.
(3) The blocks do not interpenetrate in any way.

The catalogues of spatial relations for gifts 3–6 specified according to these requirements are given in figures 10–13, respectively. In each catalogue, spatial relations are enumerated for pairs of blocks (shapes) from the appropriate vocabulary of figure 6 by examining pairs of their distinct faces. Two faces of a block are *distinct* whenever one cannot be mapped into the other via an isometry transformation that preserves the identity of the block. For example, the cube of gift 3 has one distinct face, and the oblong of gift 4 has three distinct faces. Thus in the catalogue for gift 3, spatial relations between two cubes are determined by examining a single pair of distinct faces. In the catalogue for gift 4, spatial relations between two oblongs are determined by examining six pairs of distinct faces. In general, for a pair of noncongruent blocks with m and n distinct faces apiece, it is necessary to examine mn pairs of distinct faces to determine all possible spatial relations between these blocks.

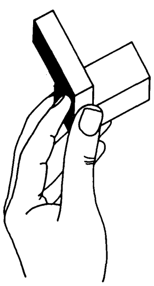


Figure 9. The formation of a spatial relation between the oblong and pillar of gift 6.

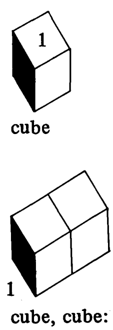


Figure 10. Spatial relation for gift 3.

The catalogues of spatial relations given in figures 10–13 correspond to the aims and intentions of the kindergarten; in the following sections, they provide all of the spatial relations used to think about designs made with the building gifts.

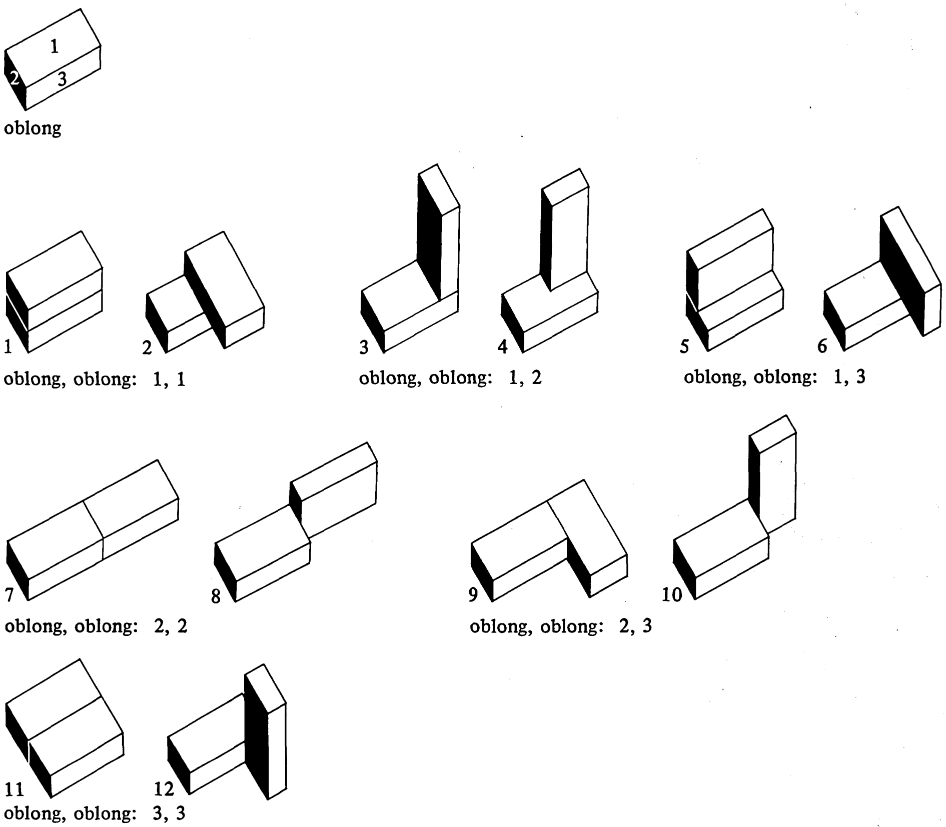


Figure 11. Spatial relations for gift 4.

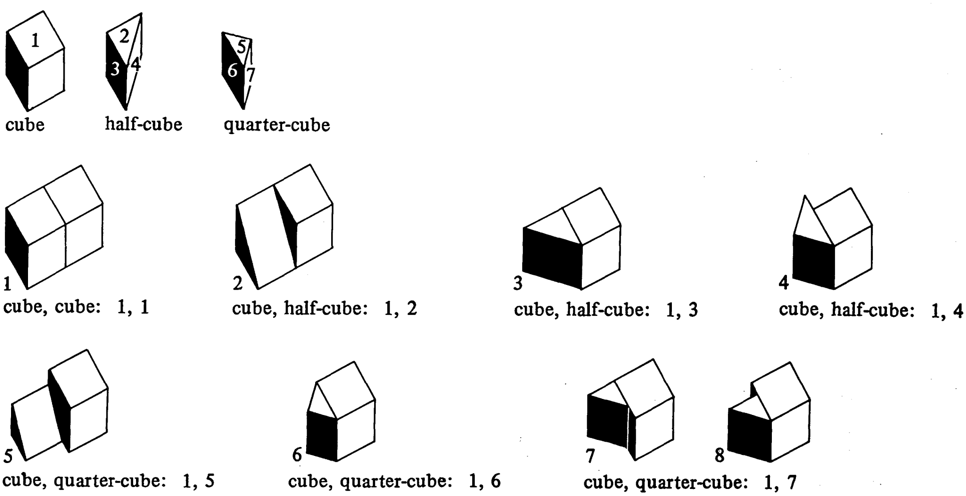


Figure 12. Spatial relations for gift 5.

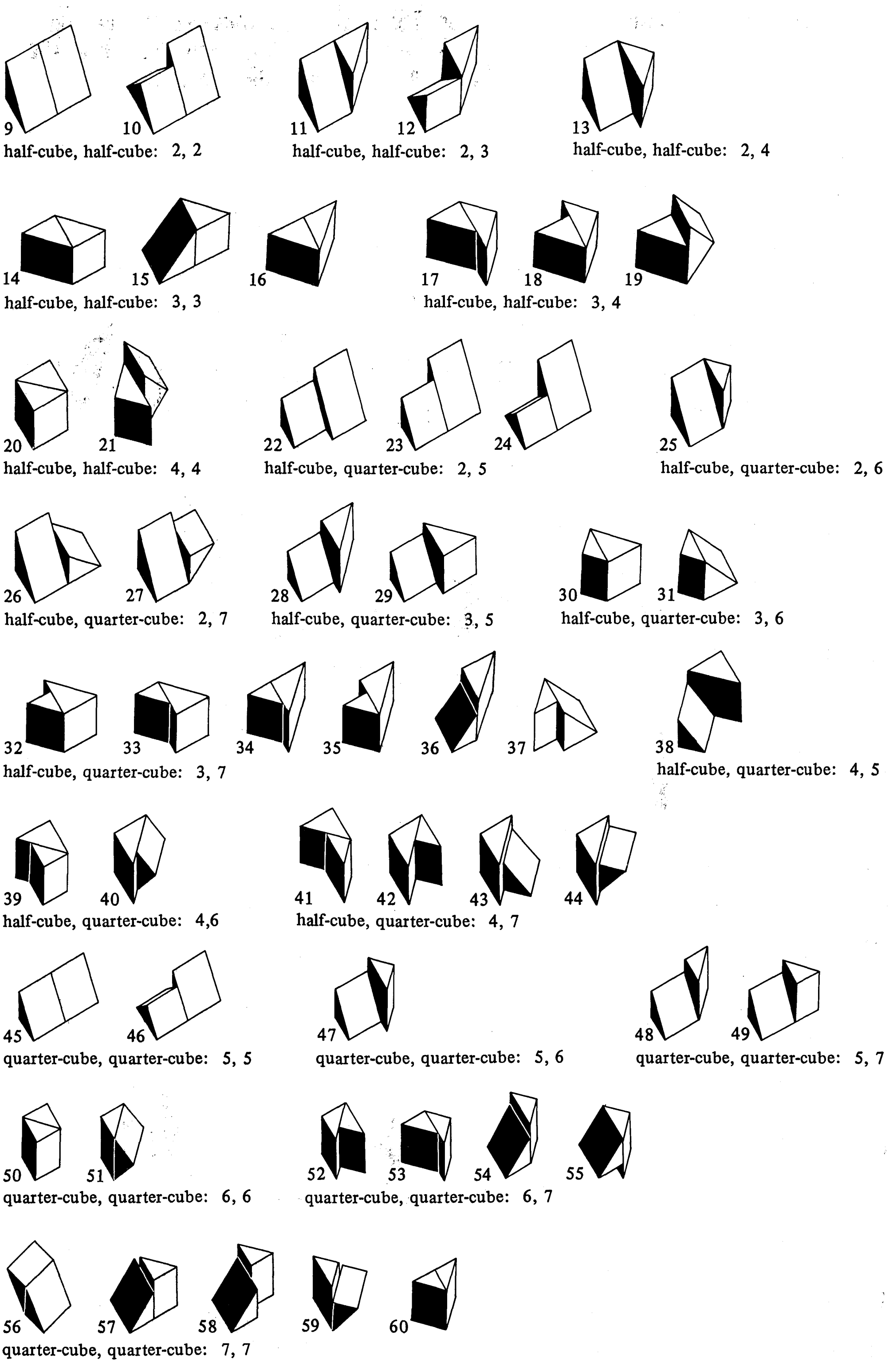


Figure 12 (continued)

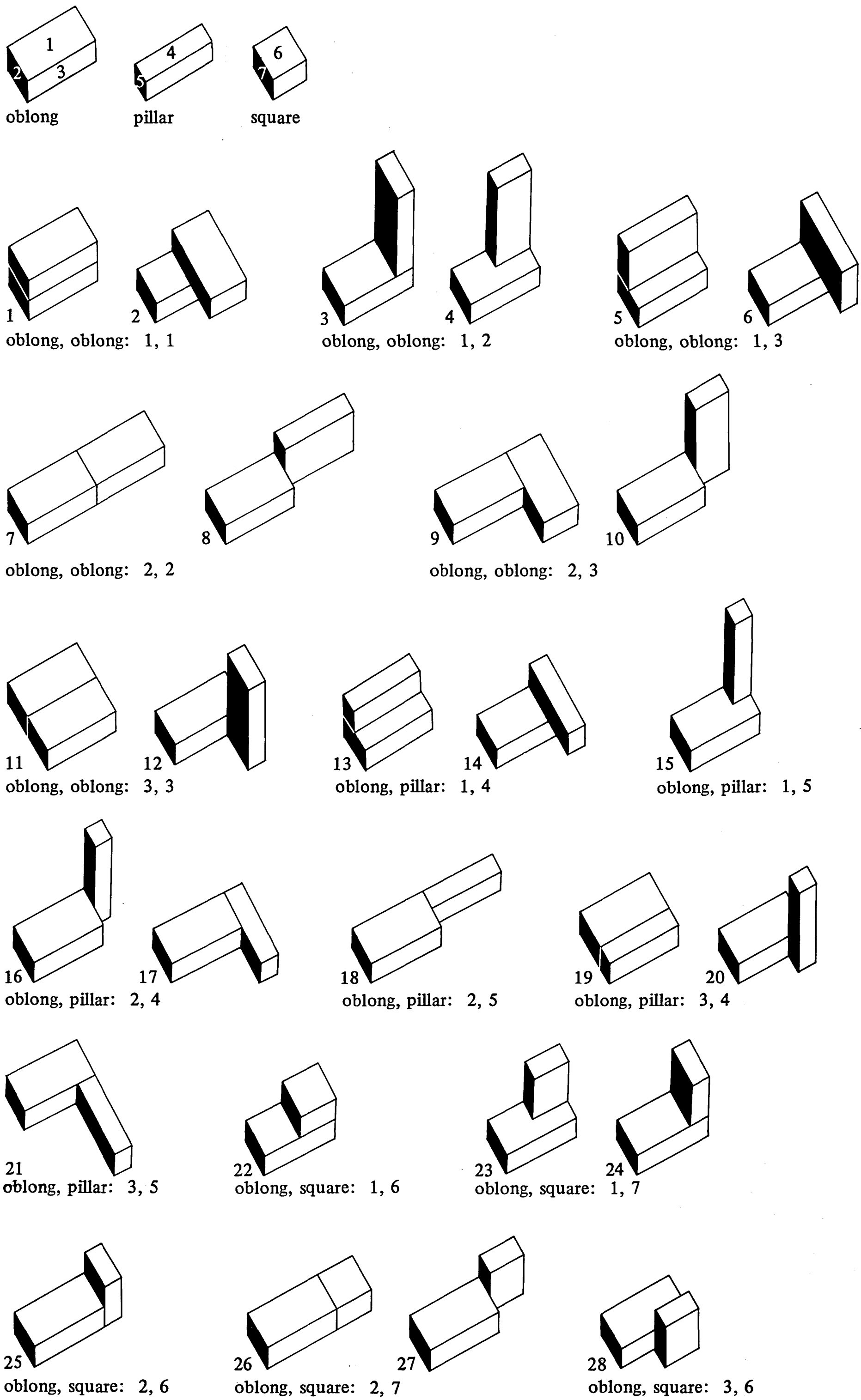


Figure 13. Spatial relations for gift 6.

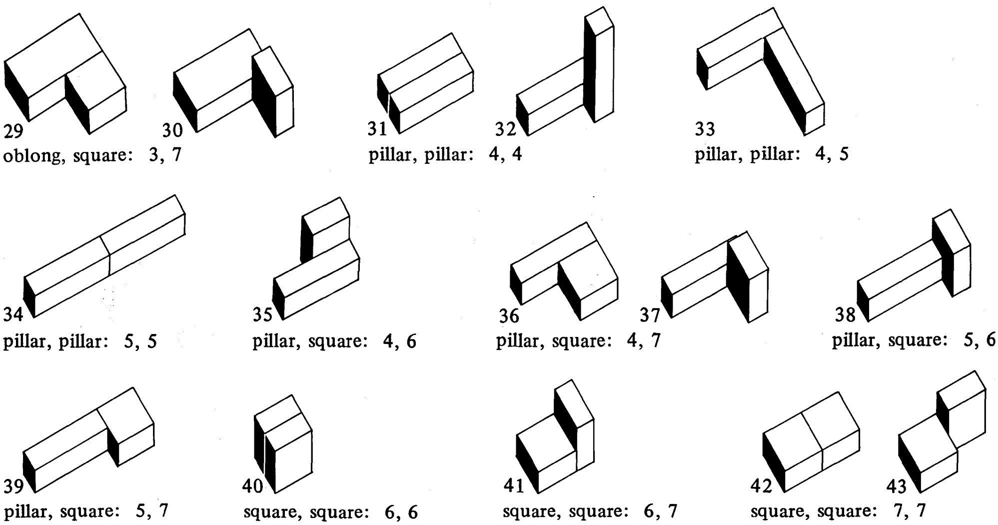


Figure 13 (continued)

Seeing designs with spatial relations

A spatial relation occurs in a design whenever parts (subshapes) of the design have the spatial relation. For example, occurrences of five spatial relations specified for gift 5 in a form of beauty made with this gift are shown in figure 14. The hyphenated index beneath each spatial relation indicates the building gift for which it is specified and its number in the catalogue of spatial relations for that gift. (This practice is also followed in subsequent figures whenever spatial relations for the building gifts are exhibited.)

Given several spatial relations, one can look for their occurrences in designs and thus direct one's seeing in a special way. For example, the form of beauty in figure 15(a) is made with the pieces of gift 4; it can be seen in one way as four squares, each formed by two oblongs having the spatial relation of figure 15(b), and

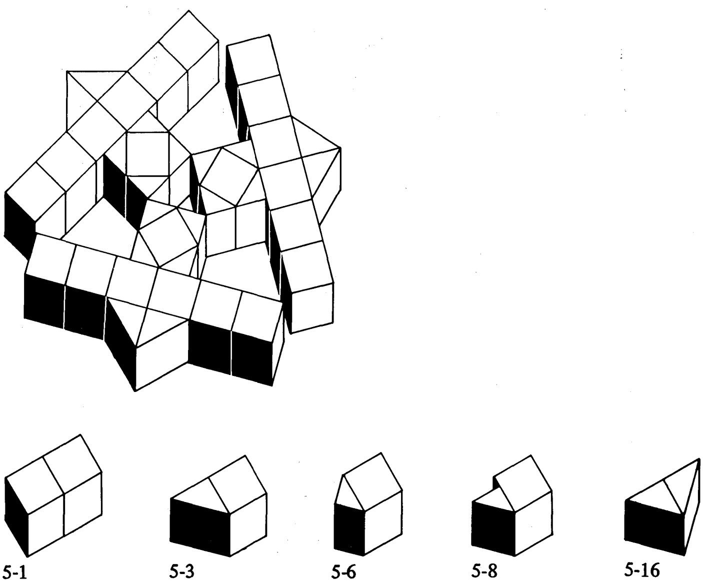


Figure 14. Spatial relations occurring in a form of beauty made with gift 5.

in an entirely different way as a swastika, with each arm formed by two oblongs having the spatial relation of figure 15(c). One's 'syntactic' understanding of a design and one's consequent 'semantic' understanding of it often depend on the vocabulary of shapes out of which it is made and the spatial relations according to which these shapes are grouped.

The spatial relations used to see designs made from a given vocabulary are determined by convention or habit. There is no fixed system of spatial relations that can be used to understand all such designs, nor is there any such design that cannot be understood in terms of a variety of different spatial relations. Ways of seeing designs may seem instinctive and invariable to the inexperienced observer; the experienced designer knows them to be acquired and variable.

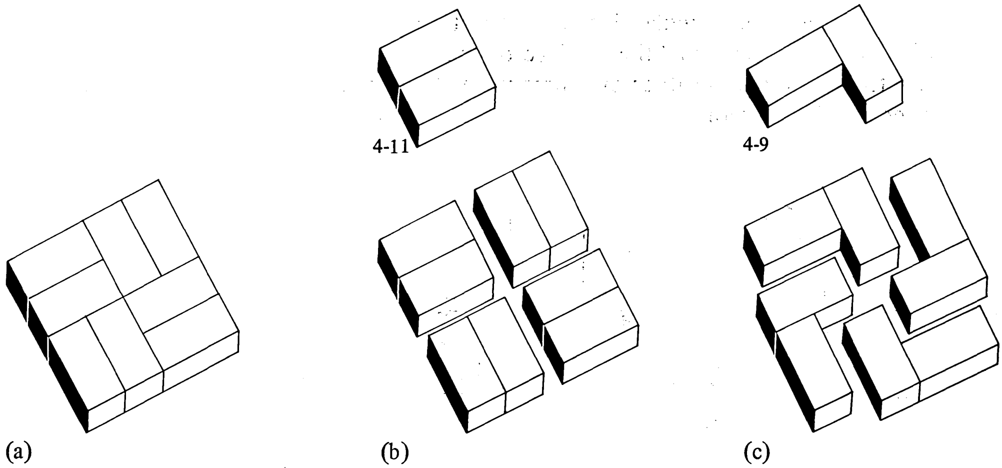


Figure 15. Different ways of seeing a form of beauty made with gift 4 according to different spatial relations.

Constructing designs with spatial relations

Designs may be understood in terms of the spatial relations actually occurring in them, or they may be understood in terms of the spatial relations used to construct them. For example, consider the designs produced in a game played by Frank Lloyd Wright and his daughter Iovanna:

> "My little five-year-old daughter Iovanna and I among constant inventions have invented a game. An architect's daughter, she has many kinds and sizes of building blocks, among them all a set of well-made cubes about an inch square painted pure bright colors, red, yellow, blue, green, black and white. Some of the cubes are divided on the diagonal into contrasting colors. Well, whoever deals, deals seven blocks to each and two diagonals. Iovanna's turn to play. She chooses a color-block—not fair to start with a diagonal—and places it square on the waxed board floor. Then I select a color-block and put it, say, touching hers at the corner. Her turn. She studies a little, head one side, finally putting down a block in whatever way she chooses, but however put down it will now make a decided change in the geometric figure. Her imagination begins to stress the judgment to decide the next play.
>
> Instead of extending the figure on the floor, she may now put a block on top of the one already 'played' by me. She does so. The figure on the floor begins to look more and more interesting as the 'third dimension' enters and the block masses creep up into the air. The group begins to be a construction. I may follow up and down, or I go crosswise with whatever color-block I may have.

But whatever I do, I will change the whole effect just as she will when her turn comes to play—change the figure—make a pattern.

Sometimes she sees she has spoiled the figure with her 'turn' and asks to change it. Always she may.

The fourteen blocks in place, we take in the result, critical or enthusiastic.

Sometimes these little form-and-color exercises in block-pattern make a good thesis in 'Modern Art'. Fact is, I intend them so" (Wright, 1932, pages 397–398).

The 'rules' of the Wrights' game can be elaborated so that the dispositions of blocks in a design are fixed by given spatial relations in an additive 'play' or in a subtractive one. These constructive processes are specified in the following recursive procedure for one or another of the building gifts and a limited number of spatial relations between its pieces:

A design begins with a single piece of the gift. A new design may be constructed from the current one by adding a piece of the gift to it or subtracting one from it, so that this piece and another one already in the current design have one of the specified spatial relations.

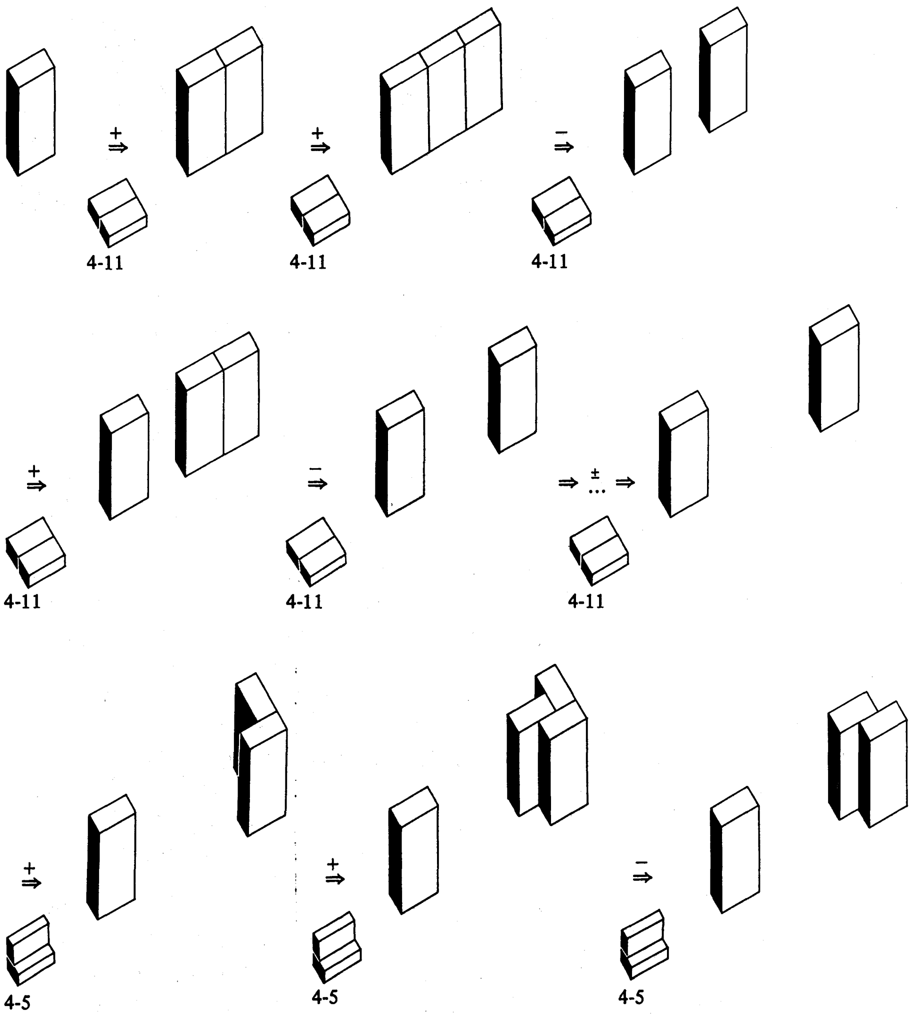


Figure 16. Construction of a form of life using the oblongs of gift 4. Spatial relations govern the placement of blocks in the design.

The application of this simple procedure is illustrated in figure 16, where it is used to construct a form of life, a stage set, with the oblongs of gift 4 according to two of the spatial relations specified for it. (Of course, this construction by itself does not fix the purpose or meaning of the design, but only its structure.) Individual designs in the construction are separated by a double arrow (⇒). The spatial relation used to go from one design to the next is shown beneath the double arrow between them. A plus (+) is placed over the double arrow whenever a piece is added according to the spatial relation, a minus (−) whenever a piece is subtracted. Notice that the spatial relations involved in subtraction provide a system of measurement for laying out the final design. Of course, the form of life constructed in figure 16 is not the only design that can be produced in this way. Each of the designs of figures 3 and 4, for example, can be constructed by means of the procedure when it is based on one or another of the building gifts and the appropriate spatial relations.

In constructions like the one of figure 16, addition allows for designs to be changed by putting in new pieces; subtraction allows for designs to be changed by taking away existing pieces. The spatial relation used for each addition thus occurs

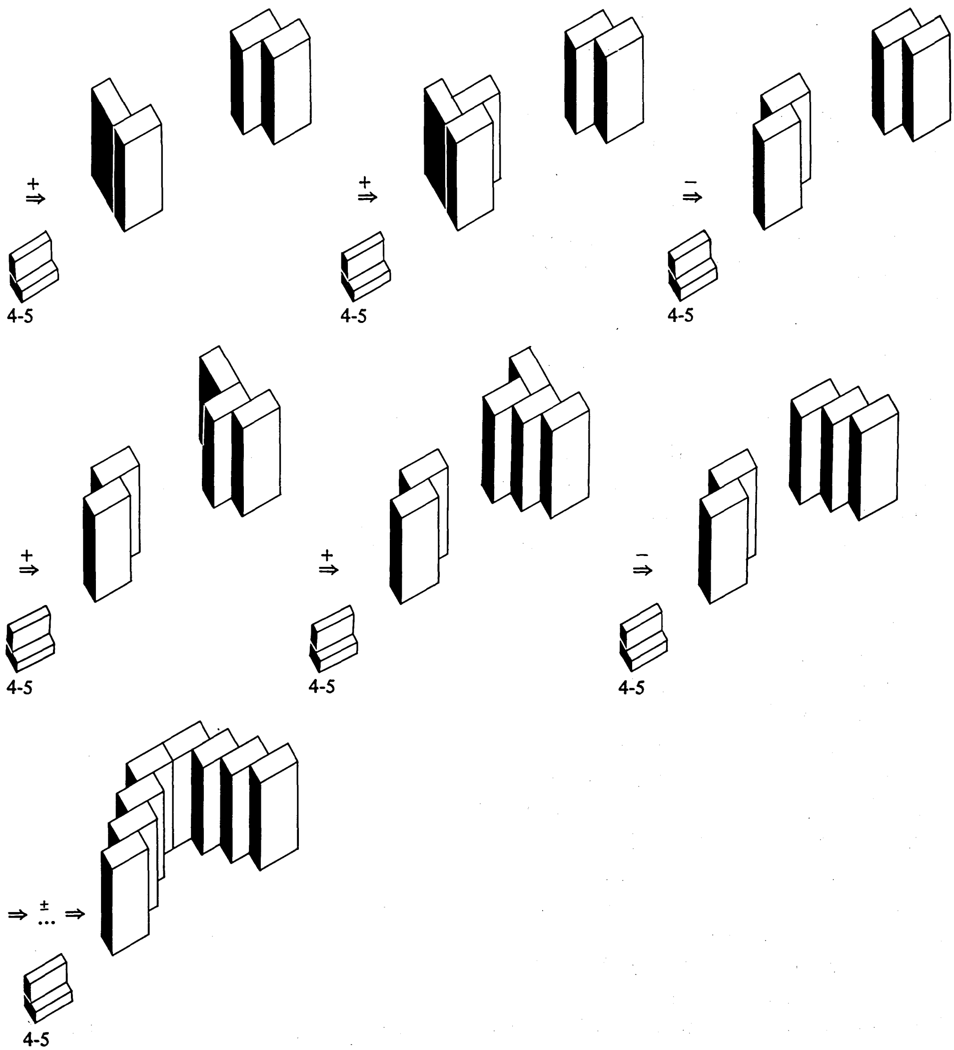


Figure 16 (continued)

in the new design; the one used for subtraction occurs in the current design. In general, the spatial relations used to construct a design may not be those actually observed in it. In this case, the latter spatial relations may be considered to be emergent ones derived from the former.

The pieces of a building gift are treated in the procedure as modules. The dimensions of these blocks fix the system of measurement and proportion for designs. Pairs of these blocks govern the arrangement of pieces in designs. In this sense, adding and subtracting building elements from a given vocabulary in terms of given spatial relations is reminiscent of building systems and modular coordination. Notice, however, that these additive and subtractive processes do not in general imply a fixed underlying grid for designs.

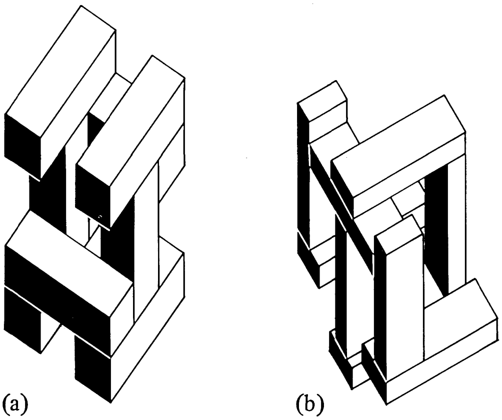
Of course, the procedure can be adapted for any vocabulary of shapes and given spatial relations. In each such case, the procedure defines a language of designs by providing the means to construct its individual members. In general, different languages of designs are defined for distinct combinations of vocabulary and spatial relations.

A language of designs defined by the procedure in terms of a given vocabulary and given spatial relations may be finite or infinite in extent. At the very least, the language contains all of the shapes in the vocabulary and one design corresponding to each spatial relation. In the latter case, designs in the language are often constructed according to multiple spatial relations specified by sets of transformations of the same two shapes, or according to spatial relations specified by pairs of shapes with some kind of symmetry, or by pairs of shapes one similar to the other or to a subshape of the other. When the procedure is based on one or another of the building gifts and the spatial relations specified for it, some combination of these criteria is met. For some examples of infinite languages of designs based on spatial relations that satisfy just the last criterion, see Stiny (1975; 1976; 1978). These languages contain designs constructed by combining similar configurations of line segments or similar polygons. Mandelbrot (1977) has defined other such languages containing designs called *fractals*.

It is interesting to observe that, in general, surprise effects or unexpected results in designs can be traced to the essential arbitrariness of the vocabulary and the spatial relations on which the procedure is based. The special structure characteristic of these designs depends on this vocabulary and these spatial relations; it emerges because the limited number of shapes in the vocabulary are combined recursively according to a limited number of spatial relations. Originality thus depends on chance events; structure follows whenever their outcomes are used over and over again to construct designs.

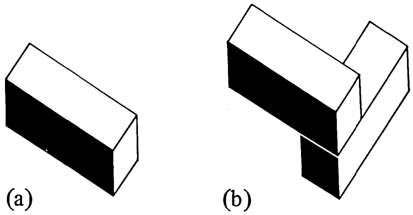
Digression

Lest the reader be concerned that seeing and constructing designs in terms of a given vocabulary of shapes and spatial relations between them is merely a game that may not be immediately applicable to 'real' design, he or she is invited to recall the constructivist designer Alexander Rodchenko's spatial constructions of 1920–1921 (for example, see Elliott, 1979, pages 44–53; Karginov, 1979, pages 77–79; and Nakov, 1975, page 25). Two representative examples of this wooden sculpture are shown in figure 17. Each one is based on a vocabulary containing a single volume, and can be seen or constructed according to spatial relations between transformations (isometries) of this shape. For example, the design of figure 17(a) is made with eight volumes like the one in figure 18(a), which are combined one with another to have the spatial relation of figure 18(b). The techniques presented in subsequent sections can be used to specify a shape grammar based on this vocabulary, and this spatial relation that defines a language containing Rodchenko's design of figure 17(a) and other new designs of the same kind. Indeed, the constructive approach outlined in this paper elaborates in very precise terms Rodchenko's intuition that "Conscious and organized LIFE, the ability to SEE and CONSTRUCT, that is the modern art" (quoted in Elliott, 1979, page 129).



(a) (b)

Figure 17. Two spatial constructions by Alexander Rodchenko.



(a) (b)

Figure 18. The vocabulary and spatial relation for the spatial construction of figure 17(a).

Shape rules

Languages of designs are defined by shape grammars with even more precision and control than the procedure described above allows. By means of the procedure, the construction of designs must begin with a single shape in a given vocabulary, and any of the given spatial relations may be used either to add or to subtract either of the shapes in occurrences of it. By means of a shape grammar, the construction of designs may begin with an arbitrary design made up of one or more shapes in the vocabulary, and any spatial relation may be used exclusively to add or to subtract just one or the other of the shapes in possibly distinguished occurrences of it. Shape grammars thus enable the full potential of vocabularies and spatial relations to be realized as a basis for constructing designs.

The shape rules in such shape grammars are defined in terms of spatial relations between shapes from any given vocabulary; they incorporate the constructive mechanisms used in the procedure.

The procedure allows for designs to be constructed by adding and subtracting shapes according to spatial relations specified by sets of shapes $\{s, t\}$. This process suggests two basic types of shape rules:

type 1: $\langle x, \varnothing \rangle \rightarrow \langle s+t, \varnothing \rangle$, type 2: $\langle s+t, \varnothing \rangle \rightarrow \langle x, \varnothing \rangle$,

where x is the shape s or x is the shape t, and $\varnothing$ is the empty set indicating that no labels are associated with shapes in either type of shape rule. Some examples of shape rules of type 1 are given in figure 19, of type 2 in figure 20. All of these shape rules are based on spatial relations specified earlier for the building gifts.

The application of shape rules of type 1 corresponds to addition in the procedure, the application of shape rules of type 2 to subtraction. In the first case, a transformation of the shape s or of the shape t is put in a design where it becomes part of an occurrence of the spatial relation specified by the set of shapes $\{s, t\}$; in the second case, this shape is taken away from a design where it is already part of an occurrence of this spatial relation. These complementary processes are illustrated in figure 21. Notice that each shape rule of type 1 or 2 allows for just one of the shapes s or t to be added to or subtracted from a design. The two shape rules of type 1 are equivalent,

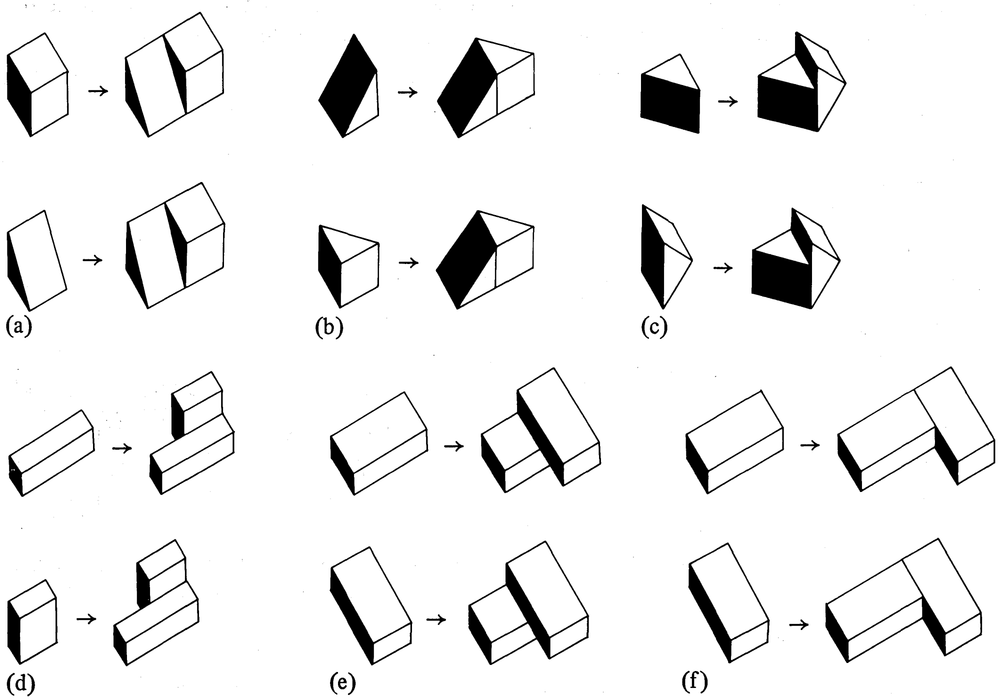


Figure 19. Some shape rules of type 1. Those in (a), (b), and (c) are based on spatial relations 5-2, 5-15, and 5-19, respectively, for gift 5; those in (d), (e), and (f) are based on spatial relations 6-35, 6-2, and 6-9, respectively, for gift 6.

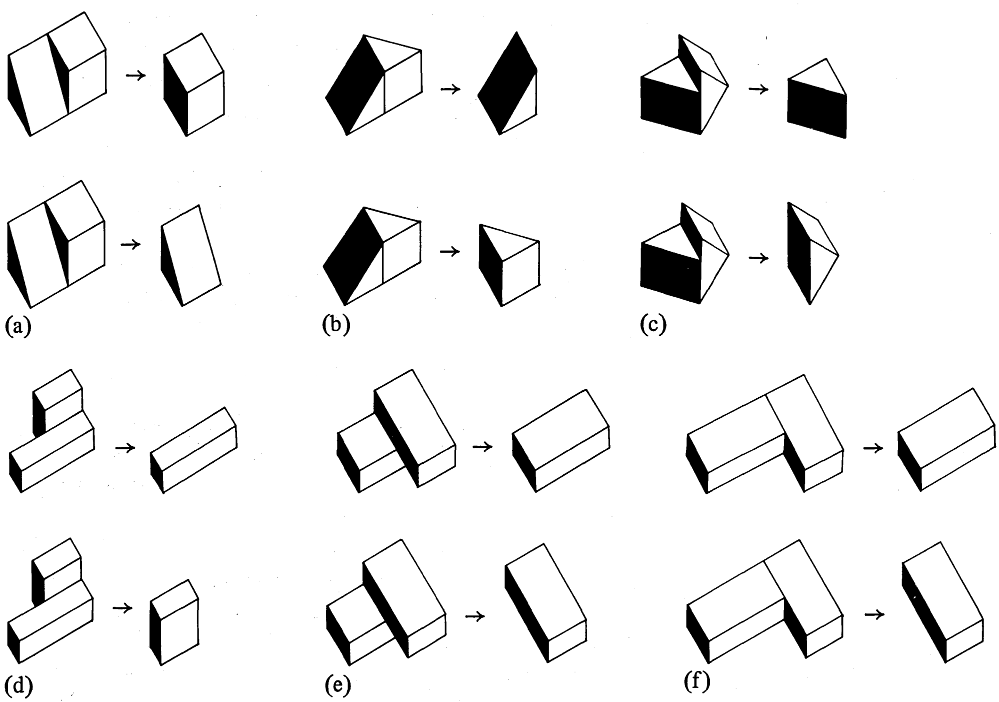


Figure 20. Some shape rules of type 2 defined for spatial relations as in figure 19.

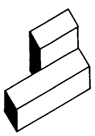
and so are the two shape rules of type 2 whenever the shapes s and $s+t$ and the shapes t and $s+t$ have the same spatial relation. In this case, s and t are congruent and $s+t$ has some kind of symmetry. For example, the two shape rules of type 1 given in figure 19(b) are equivalent, as are the two shape rules of type 1 given in figure 19(e); the two shape rules of type 2 given in each of the figures 20(b) and (e) are also equivalent. As will be seen shortly, for technical reasons pertaining to the way shape rules apply, the correspondence between shape rules of type 2 and subtraction is not always exact.

The application of shape rules of types 1 and 2 can be controlled in various ways by labelling the shapes occurring in them. Shape rules of two more general types are thus defined:

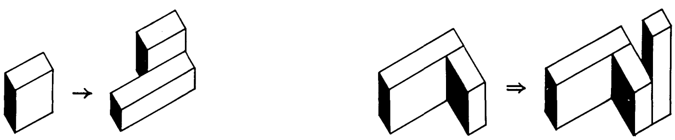
type 3: $\langle x, P\rangle \rightarrow \langle s+t, Q\rangle$, type 4: $\langle s+t, P\rangle \rightarrow \langle x, Q\rangle$,

where x, s, and t are as before, and P and Q are sets of labelled points. In shape rules of these new types, special ways of adding and subtracting shapes according to spatial relations may be distinguished to construct designs. Some simple labelling schemes that can be employed profitably to obtain shape rules of types 3 and 4 from shape rules of types 1 and 2 are elaborated below.

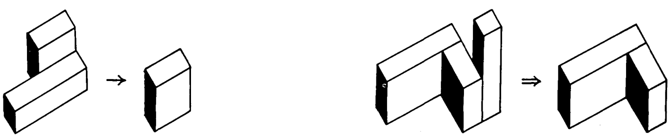
Before we turn to these schemes, however, it is important to observe that shape rules of types 3 and 4 provide the usual basis of 'writing' and 'erasing' required for general (Turing) computation. Indeed, it can be shown without too much trouble that the construction of designs in any language defined by a shape grammar can be considered in terms of a finite collection of spatial relations specified by sets of shapes $\{s, t\}$ depending only on the shape grammar, and shape rules of types 3 and 4 defined from these spatial relations. Thus by focusing one's attention on shape rules of types 3 and 4, one does not restrict in any way the kinds of designs one can construct or the languages of designs one can define.



6-35
(a)



shape rule of type 1 shape rule application
(b)



shape rule of type 2 shape rule application
(c)

Figure 21. Application of shape rules of types 1 and 2 based on the same spatial relation.

Symmetry and shape rule application

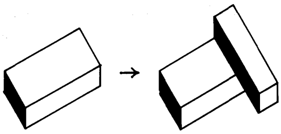
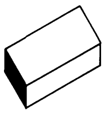
The first labelling scheme we consider allows for the sequence of shape rule applications used to construct designs to be controlled in terms of the symmetry properties of the shapes occurring in shape rules.

The number of distinct transformations under which a shape rule $\alpha \rightarrow \beta$ can be applied to a particular part of a design depends on the symmetry properties of its left-hand side α. More precisely, if the symmetry group of α is of order n, then the shape rule $\alpha \rightarrow \beta$ can be applied to each subshape of a design similar to α in exactly n distinct ways.

For example, consider the shape rule of type 1 given in figure 22(a). The symmetry group of the left-hand side of this shape rule has order eight. Consequently, the shape rule can be applied to the design consisting of the single oblong shown in figure 22(b) in eight distinct ways, as indicated in figure 22(c). The symmetry group of the left-hand side of the shape rule of type 2 given in figure 23(a) has order two. This shape rule can be applied to the design made up of the pair of oblongs shown in figure 23(b) in two distinct ways as indicated in figure 23(c). Notice that in general, the symmetry groups of the left-hand sides of shape rules of type 2 have order one.

The distinct applications of a shape rule $\alpha \rightarrow \beta$ can be distinguished by labelling its left-hand side. By associating a symbol with α at a point that is not mapped into itself via any element in the symmetry group of α but the identity, the symmetry of α is destroyed utterly. The manner in which the new shape rule $\alpha' \rightarrow \beta$ so defined applies to any subshape of a design similar to α' thus depends on the location of this symbol in the subshape.

For example, the left-hand side of the shape rule of type 1 given in figure 22(a) can be labelled in this way as shown in figure 24(a). The shape rule of type 3 so defined applies to each of the eight labelled oblongs given in figure 24(b) in exactly one way as illustrated in figure 24(c). The designs of figure 24(b) are produced by


(a)

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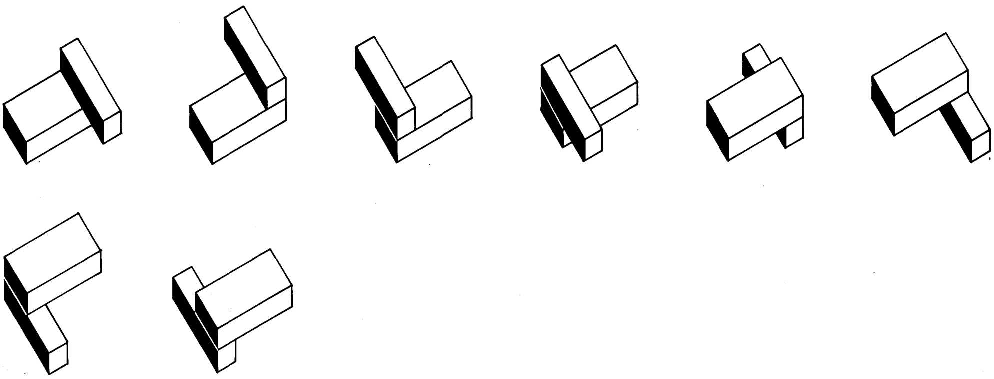
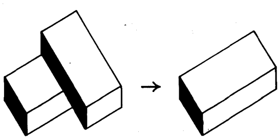
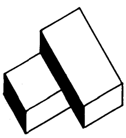

(c)

Figure 22. Applications of a shape rule of type 1 under different transformations.


(a)

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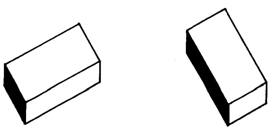

(c)

Figure 23. Applications of a shape rule of type 2 under different transformations.

applying the elements in the symmetry group of the left-hand side of the original shape rule of figure 22(a) to the left-hand side of the new shape rule of figure 24(a). (In figure 24 and subsequent figures, labels associated with a front face of a volume are closed; those associated with a back face are open. Thus, for example, in figure 24 • and ○ are two versions of the same symbol: • is associated with a front face of an oblong, and ○ is associated with a back face of an oblong.) In the same way, a shape rule of type 4 is defined from the shape rule of type 2 given in figure 23(a) as shown in figure 25(a). This new shape rule applies to each of the two designs of figure 25(b) in exactly one way as illustrated in figure 25(c). Here again, the designs of figure 25(b) are produced by applying the elements in the symmetry group of the left-hand side of the original shape rule of figure 23(a) to the left-hand side of the new shape rule of figure 25(a). Notice that the shape rule of type 1 in figure 24(a)

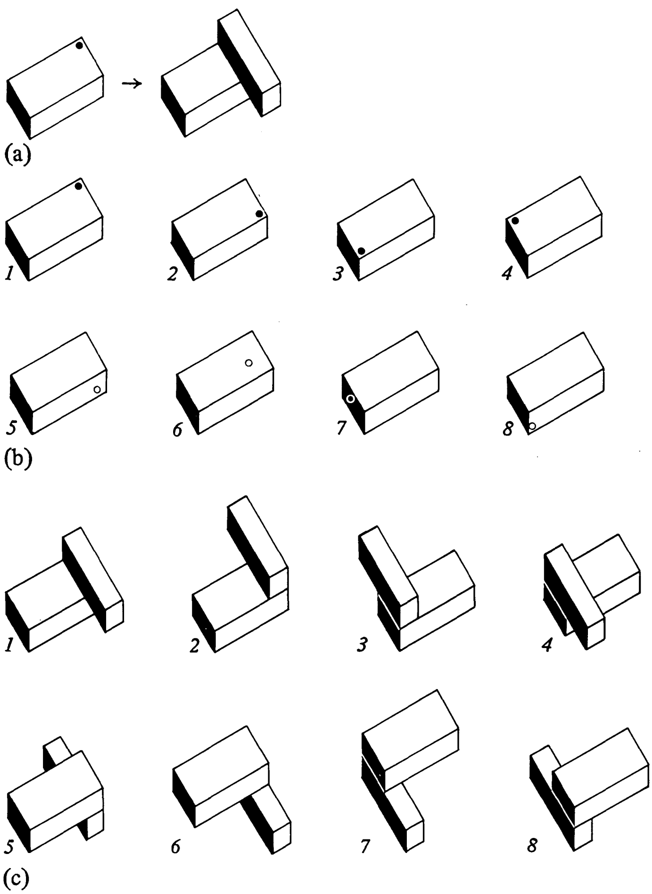


Figure 24. Applications of the shape rule of figure 22 labelled according to the symmetry group of the oblong in its left-hand side.

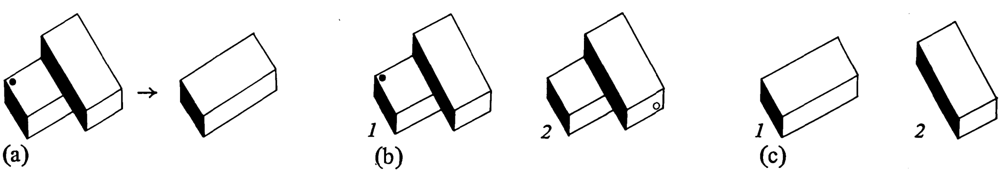


Figure 25. Applications of the shape rule of figure 23 labelled according to the symmetry group of its left-hand side.

and the one of type 2 in figure 25(a) both erase the symbol associated with their left-hand sides whenever they are applied to a design.

In general, the left-hand side of a shape rule $\alpha \rightarrow \beta$ can be labelled in a variety of different ways to distinguish its distinct applications. Finding one of these labellings is usually straightforward once the symmetry group of α is known. For example, when the pieces of the building gifts occur in the left-hand sides of shape rules of type 1, they can be labelled as shown in figure 26. The symbol • is associated with a rectangular face of each piece that has maximal area. It is assumed that this symbol is not located on the face but just under it, so that no confusion results when two blocks share a face or part of one. The order of the symmetry group for each piece is given beneath it.

Now suppose that the application of a shape rule $\alpha \rightarrow \beta$ to a design depends on α being similar to subshapes of the right-hand sides of some of the shape rules used to construct the design, as is generally the case with shape rules of types 1 and 2. If the distinct applications of this shape rule are to be distinguished by labelling α according to its symmetry group, then these subshapes must be labelled to correspond to one or more distinct transformations of the labelled version α' of α. More specifically, each label associated with one of these subshapes must produce a labelled shape that is similar to α'. Thus if the order of the symmetry group of α is n, as many as n labels can be associated with a subshape, and this subshape can be labelled in 2^n distinct ways. Each label determines a particular application of the shape rule $\alpha' \rightarrow \beta$.

These considerations are illustrated by the two shape rules of type 3 given in figure 27(a). The oblong in the left-hand side of the first shape rule is labelled according to its symmetry group. This shape in the right-hand side of the second

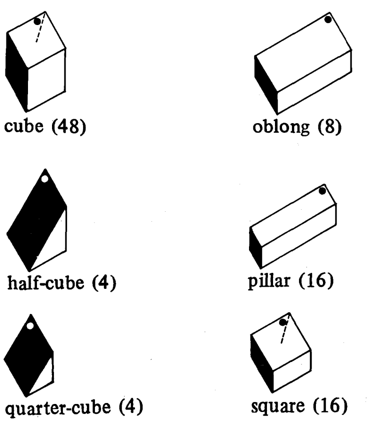


Figure 26. Canonical labelling of the pieces of the building gifts according to their symmetry groups.

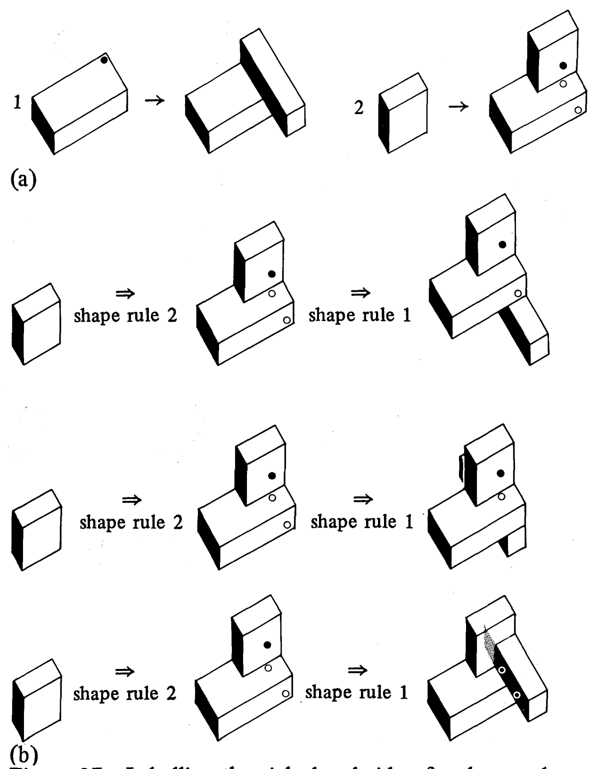


Figure 27. Labelling the right-hand side of a shape rule of type 1.

shape rule is labelled to correspond to three transformations of the left-hand side of the first shape rule. Because the order of the symmetry group of the oblong is eight, its occurrence in the right-hand side of the second shape rule could have as many as eight labels associated with it. Each label determines a particular application of the first shape rule. The three applications actually determined by the second shape rule after it has been used to construct a design are shown in figure 27(b). Notice that in the third application, the pillar interpenetrates the square.

Of course, both sides of shape rules of types 1 and 2 can be labelled in a coordinated way in terms of the symmetry properties of the shapes in their left-hand sides to obtain shape rules of types 3 and 4. In particular, consider a collection of shape rules of type 1 based on multiple spatial relations between the shapes in a given vocabulary. If occurrences of these shapes in the left-hand sides of shape rules have a

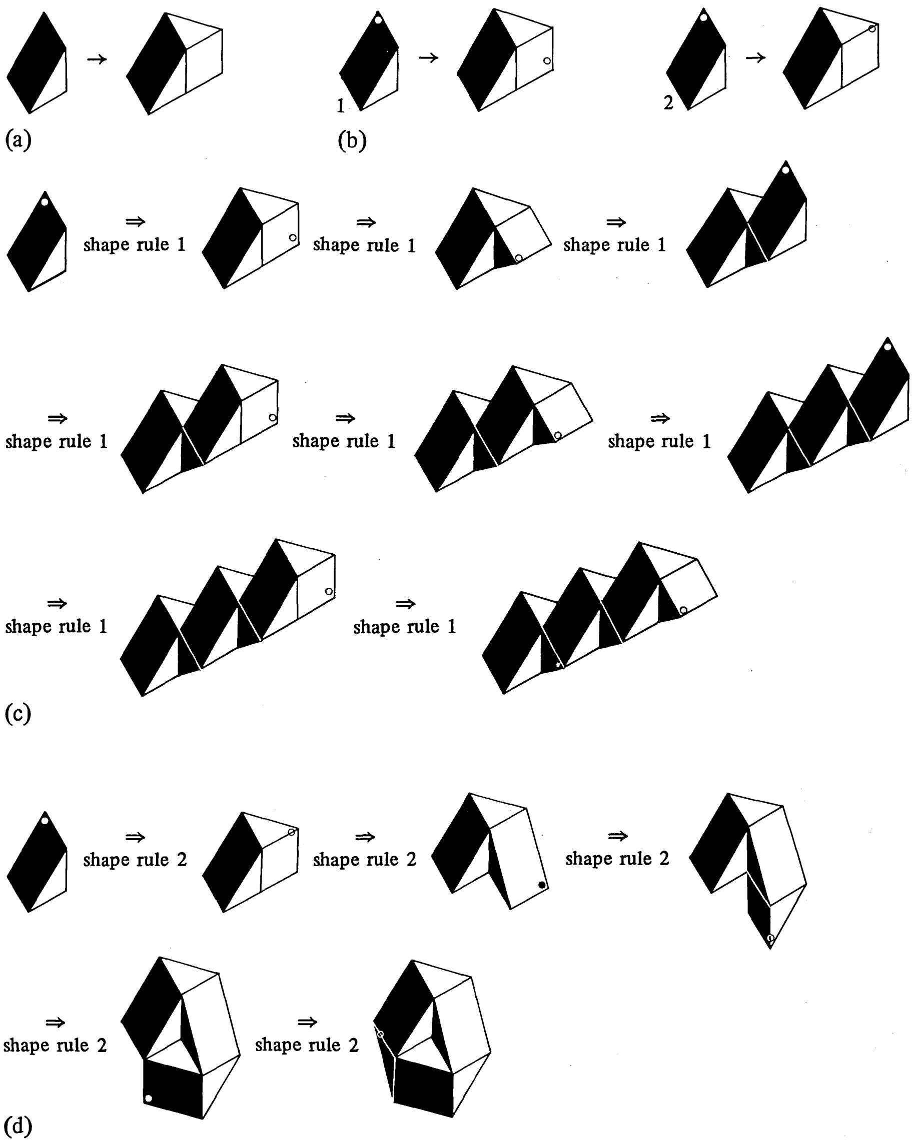


Figure 28. Designs constructed by labelling a shape rule of type 1 in two different ways according to the symmetry group of a half-cube.

fixed symbol associated with them to destroy their symmetry, then their occurrences in the right-hand sides of shape rules can be labelled to correspond to multiple transformations of their labelled versions. There are thus $2^m \cdot 2^n$ shape rules of type 3 that can be obtained in this way from each shape rule of type 1 comprised of shapes with symmetry groups of orders m and n. [In the more general case where r ($\geqslant 1$) different symbols are used for labelling, there are $r \cdot (r+1)^m \cdot (r+1)^n$ shape rules of type 3.] Such shape rules can be used to construct strikingly different designs, each having different symmetry properties, but all based on the same spatial relations. For example, a spatial relation between two half-cubes provides the basis for the shape rule of type 1 given in figure 28(a). Two ways of labelling this shape rule according to the symmetry group of the half-cube are shown in figure 28(b). Both of these shape rules are of type 3; they can be applied recursively to construct two different designs as illustrated in figures 28(c) and (d). In the first construction, the shape rule is applied under rotations and translations; in the second construction, the shape rule is applied under reflections, rotations, and translations. Each of these designs has different symmetry properties, but both are based on the same spatial relation. Later in this paper we consider languages of designs defined by shape grammars using shape rules of types 3 and 4 obtained from shape rules of types 1 and 2 by labelling them in terms of the symmetry properties of the shapes occurring in their left-hand sides.

Spatial ambiguity

Spatial relations for a given vocabulary sometimes allow for the construction of designs with subshapes that are shapes in the vocabulary but not the ones actually combined to make the designs. For example, the spatial relation for gift 3 allows for cubes to be constructed from cubes. Shape rules of types 1 and 2 based on such spatial relations may be applied to construct designs with certain kinds of spatial ambiguity.

Consider the sequences of shape rule applications in figure 29. All of the shape rules used in these sequences are based on one or another of the spatial relations for the building gifts. In each sequence, the last shape rule applies to part of a design

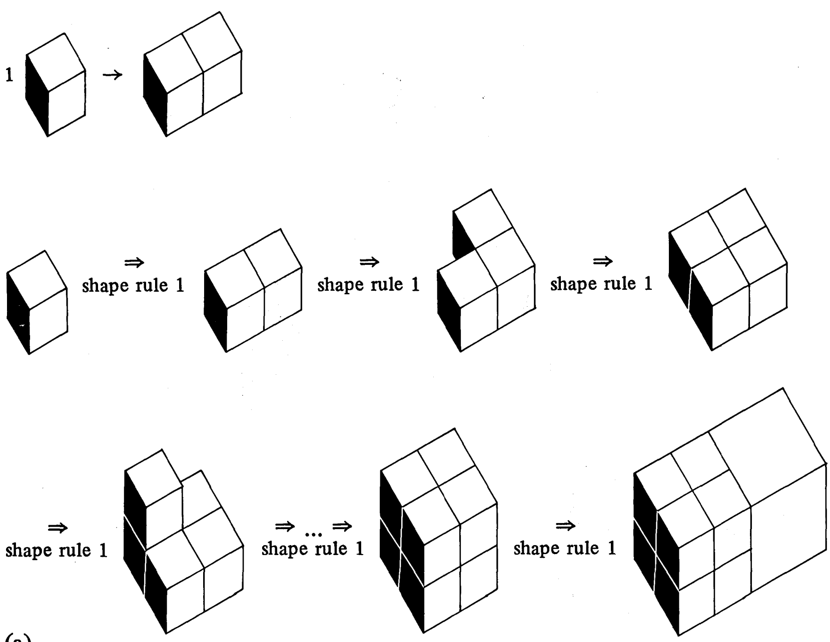


Figure 29. Spatial ambiguity in the construction of designs.

that is similar to a piece in one of the building gifts formed by combining other pieces in the gift.

The first sequence is based on gift 3. Eight cubes are combined to make a larger cube to which another cube of this size is then added according to a spatial relation

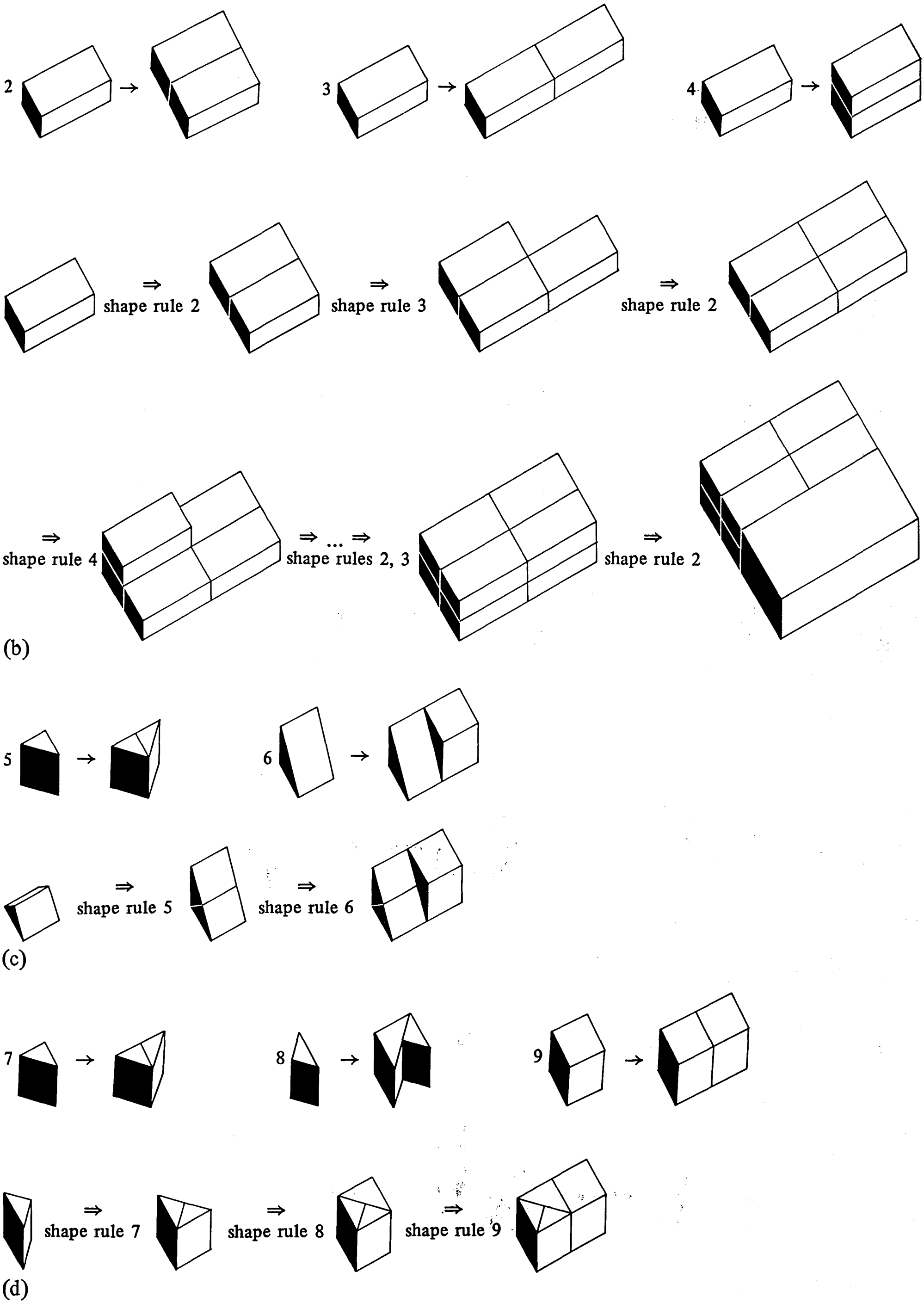
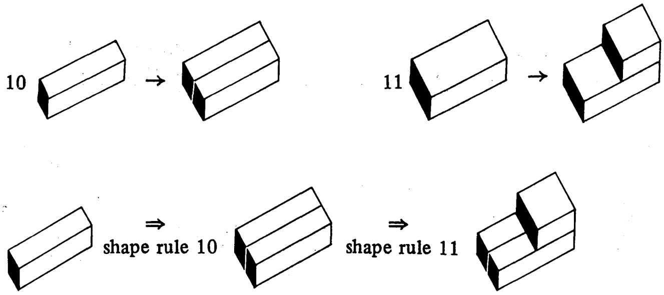
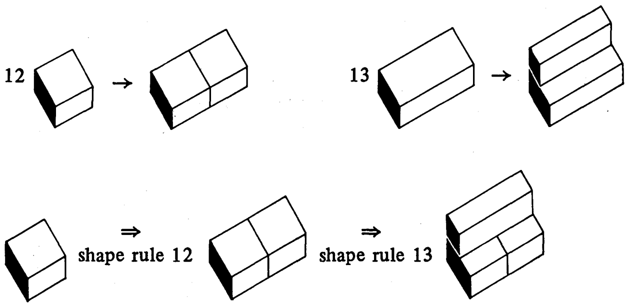


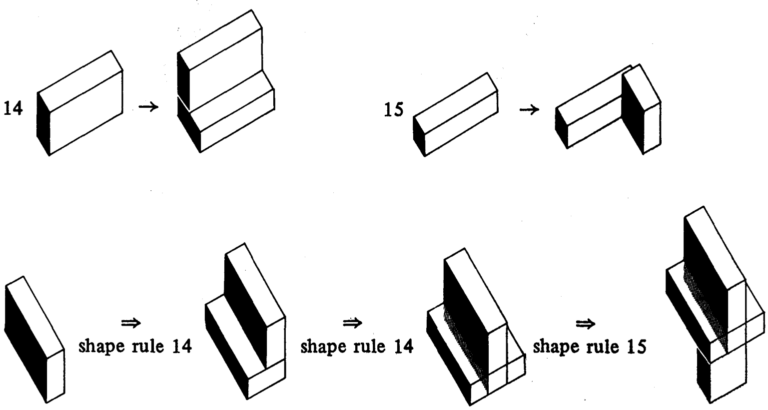
Figure 29 (continued)



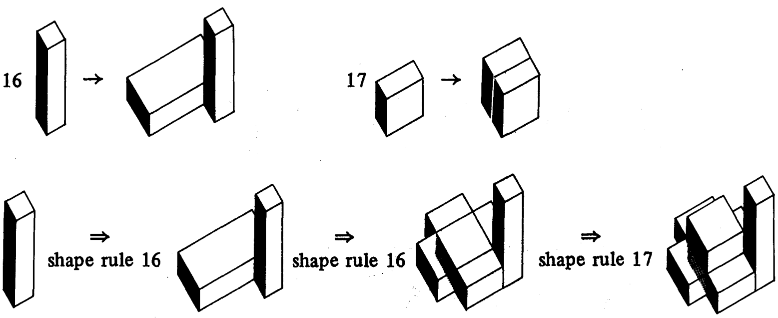
(e)



(f)



(g)



(h)

Figure 29 (continued)

between two cubes. The second sequence is based on gift 4. Eight oblongs are combined to make a larger oblong to which another oblong of this size is then added according to a spatial relation between two oblongs. The third and fourth sequences are based on gift 5. In the third, two quarter-cubes are combined to make a half-cube to which a cube is then added according to a spatial relation between a half-cube and a cube; in the fourth, two quarter-cubes and a half-cube are combined to make a cube to which a cube is then added according to a spatial relation between two cubes. The fifth through eighth sequences are based on gift 6. In the fifth, two pillars are combined to make an oblong to which a square is then added according to a spatial relation between an oblong and a square; in the sixth, two squares are combined to make an oblong to which a pillar is then added according to a spatial relation between an oblong and a pillar; in the seventh, two oblongs interpenetrate to make a pillar to which a square is then added according to a spatial relation between a pillar and a square; and in the eighth, two oblongs interpenetrate to make a square to which another square is then added according to a spatial relation between two squares.

Spatial ambiguities like those illustrated in figure 29 are sometimes exploited in designs with considerable visual effect; other times, they result in confusion and are avoided. In the latter case, shape rules of types 1 and 2 can be labelled so that they apply only to the shapes actually combined in designs. Such labelling schemes usually depend on the specific geometry of the shapes in the spatial relations on which these shape rules are based.

In cases where scale is involved as in the first and second sequences of figure 29, spatial ambiguity can be prohibited by requiring that shape rules apply under isometries only. Alternatively, the individual shapes in the left-hand and right-hand sides of shape rules can be labelled as in, for example, figure 30. Here, the individual cubes in the left-hand and right-hand sides of the shape rule of type 1 used in the first sequence are labelled. Two symbols are located on each diagonal of a cube at a fixed distance from its centre and connected by a straight line. No arrangement of cubes labelled in this way can have a subshape that is also a cube labelled in this way, which is not one originally combined in the arrangement. Hence, the shape rule of type 3 so defined can be applied only to cubes actually combined to make a design. Notice that this labelling scheme preserves the symmetry of a cube. The oblong in the shape rules of type 1 used in the second sequence can be labelled similarly to define shape rules of type 3.

In cases where scale is not involved as in the third through sixth sequences of figure 29, spatial ambiguity can be prohibited by labelling the individual shapes in the left-hand and right-hand sides of shape rules as in, for example, figure 31. Here, a symbol is located at the centroid of the half-cubes in the left-hand and right-hand sides of the shape rule of type 1 applied at the end of the third sequence. As a result, this shape rule of type 3 cannot now be applied to a half-cube formed by two quarter-cubes.

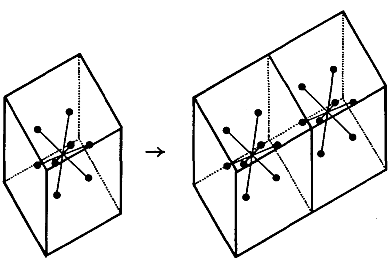


Figure 30. Labelling a cube to prevent spatial ambiguity. (Scale ×2.)

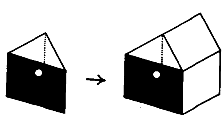


Figure 31. Labelling a half-cube to prevent spatial ambiguity.

Schemes of this kind can also be used to label the appropriate shape rules in the fourth through sixth sequences. Notice that such schemes may not be appropriate when scale or interpenetration is involved. For example, consider a $3 \times 3 \times 3$ cube made up of unit cubes with symbols at their centroids or the interpenetration between the two oblongs in the eighth sequence when oblongs as well as squares have symbols at their centroids.

Avoiding spatial ambiguities caused by interpenetration is often very tricky. In general, each case must be treated separately. In the seventh and eighth sequences of figure 29, for example, either of the labelling schemes described above can be used.

Shape rules of types 1 and 2 can be labelled selectively to allow some kinds of spatial ambiguity and to disallow other kinds. Labelling provides considerable choice in deciding how shape rules are to be applied and thus what kinds of designs are to be constructed.

Interpenetration
As has already been seen in figures 27 and 29, spatial relations for a given vocabulary sometimes allow for its shapes to interpenetrate in designs. Some more examples of interpenetration are given in figure 32. Shape rules based on the spatial relations for gift 6 are used to construct these designs.

Interpenetration like spatial ambiguity can be a valuable technique in design. Architects and designers often conceive of designs in terms of interpenetrating masses or volumes. In those cases, however, where interpenetration is felt to be undesirable, it can be prohibited by labelling the shape rules used to construct designs.

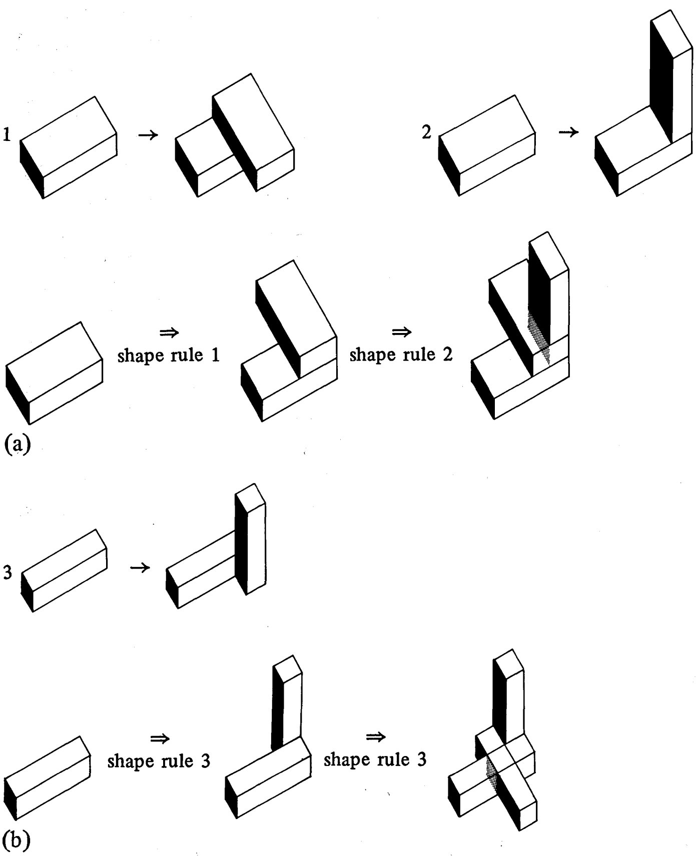


Figure 32. The construction of designs in which volumes interpenetrate.

In this section, a simple labelling scheme is developed to prevent interpenetration in designs constructed by shape rules applied under isometries. This scheme is used to label shape rules of types 1 and 2 based on spatial relations that can be specified by sets of shapes defined in a discrete, cubical grid, for example, the spatial relations for gifts 3, 4, and 6. The scheme can be used in more general cases, for example, for gift 5, by approximating spatial relations in such grids.

Assume that designs based on such spatial relations are constructed in the cubical grid corresponding to the shapes used to specify them. For the empty spaces in this grid to influence the application of shape rules, they must be identified explicitly. Whether a shape rule applies to a design depends entirely on the lines and symbols occurring in it. The absence of these elements cannot be recognized. Thus each cell in the grid not occupied by any part of a design must be labelled. For example, in

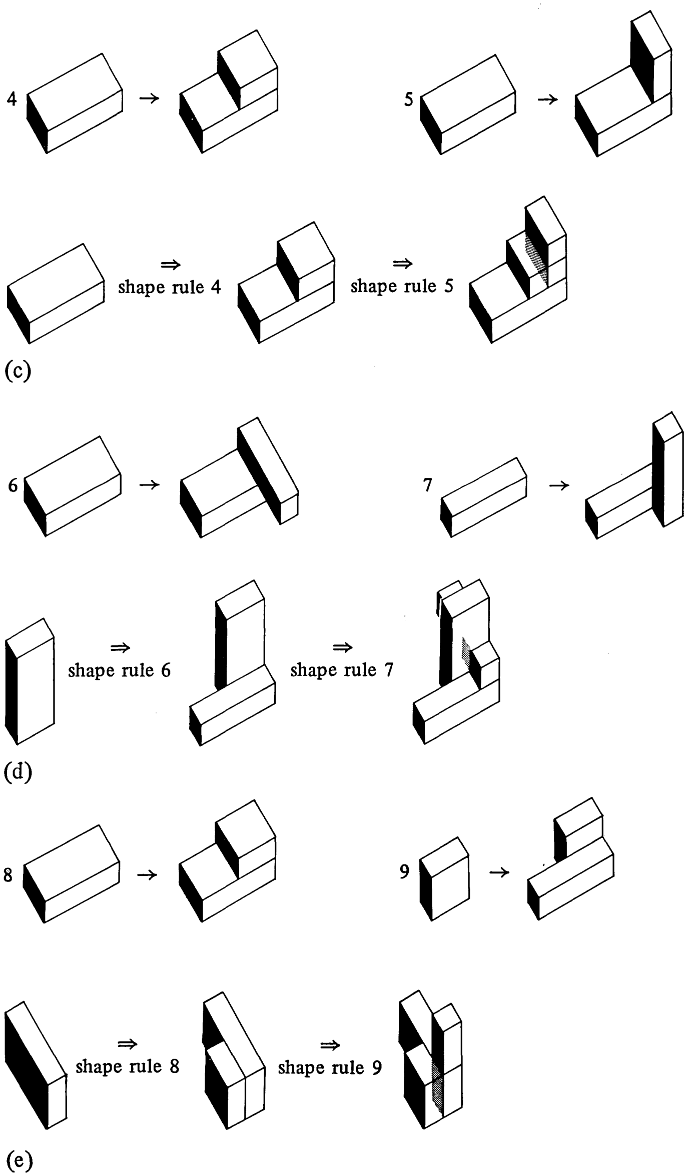


Figure 32 (continued)

figure 33 the cells in the grid for gift 6 not occupied by the design consisting of a single oblong have a special symbol at their centroids. (This symbol can be located in appropriate cells by applying shape rules which are most conveniently defined in a parametric shape grammar.)

If interpenetration is not allowed, a shape rule of type 1 can be applied to add a shape to a design only when the shape is to occupy labelled cells. These cells can be indicated in the left-hand side of the shape rule by the symbols at their centroids. Thus, for example, the shape rule of type 1 in figure 34(a) is labelled as shown in figure 34(b) to define a shape rule of type 3. This new shape rule applies to an oblong that is part of a design constructed in the grid. A pillar is added only when labelled cells corresponding to the pillar are associated with the oblong according to the spatial relation used as the basis for the shape rule. When the shape rule is applied, the symbols at the centroids of the cells corresponding to the pillar are erased. As a result, no shape rule labelled in this way can now be applied to add another shape that interpenetrates the pillar. Notice that open spaces may be associated with some of the shapes combined in a design by treating these spaces as parts of these shapes when they are added during the construction of the design. The cells in these spaces can be labelled by a new symbol so that they can be associated with more than one shape combined in the design.

Whenever a shape rule of type 2 applies to subtract a shape from a design constructed in the grid, it leaves a space that can be filled later by another shape or combination of shapes. This space can be indicated in the right-hand side of the shape rule by putting in symbols at the centroids of the cells corresponding to it. Thus, for example, the shape rule of type 2 in figure 35(a) is labelled as shown in figure 35(b) to define a shape rule of type 4.

Shape rules of types 1 and 2 can be labelled selectively to allow some kinds of interpenetration and to disallow other kinds. Interpenetration in designs can thus be controlled with considerable precision.

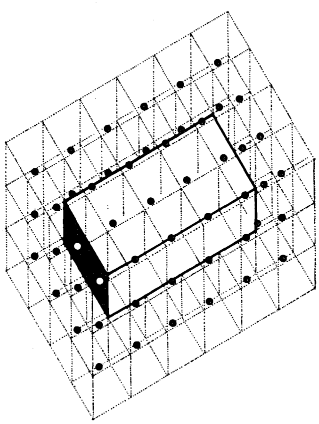


Figure 33. Labelling the cells in a cubical grid to indicate empty spaces. Labels occluded by the oblong in the grid are not shown. The oblong in the grid coincides with its lines and is located one grid cell away from each of its faces. (Scale ×2.)

(a) (b)

Figure 34. Labelling a shape rule of type 1 to apply in a labelled cubical grid.

Labelling schemes restrict the application of shape rules of types 1 and 2 to define shape rules of types 3 and 4. As a result, designs with special properties involving symmetry, spatial ambiguity, and interpenetration can be constructed. Of course, shape rules of these four basic types can be elaborated further so as to construct designs with other special properties. For example, such shape rules can be combined and parameterized to obtain shape rule schemata that apply in the construction of designs with bilateral symmetry with respect to one or more axes or with more general symmetry properties fixed by a limited number of rotations and reflections, as described by Stiny and Mitchell (1978; 1980). The techniques developed in this paper only begin to suggest possibilities for shape rules and shape rule schemata that can be used to construct designs based on given vocabularies and spatial relations between the shapes in them. Continued research in this area should lead to a host of general techniques for defining languages of designs.

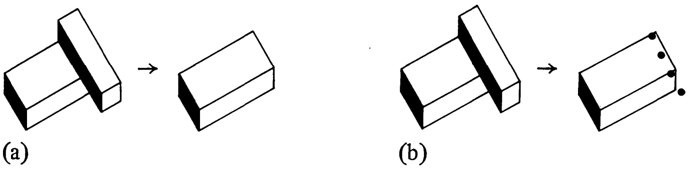


Figure 35. Labelling a shape rule of type 2 to apply in a labelled cubical grid.

Shape rules of type 2

A shape rule of type 2 is applied to a design to subtract a shape in an occurrence of a given spatial relation. Whenever some of the lines defining this shape overlap lines in one or more shapes combined in the design that are not in this occurrence of the spatial relation, the application of the shape rule erases parts of the design that should not be erased.

For example, consider the application of the shape rule of type 2 shown in figure 36. Here, the shape rule is based on the spatial relation between two cubes specified for gift 3; it is applied to a design consisting of three cubes to subtract the central one, as this cube and the left-most one have the required spatial relation. Because the central cube shares a face with the right-most one, this face is also erased, thus producing an incomplete design. In order to avoid this unpleasantness, shape rules of type 2 must be applied in conjunction with other shape rules that replace missing parts of designs.

One way to avoid incomplete designs resulting from applications of shape rules of type 2 is to label them and to use the resulting shape rules of type 4 in parametric

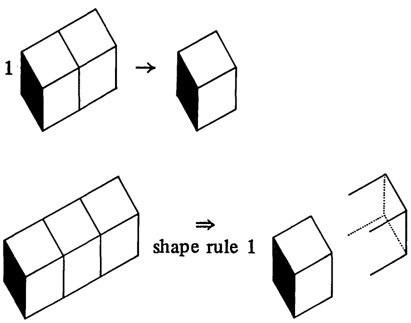
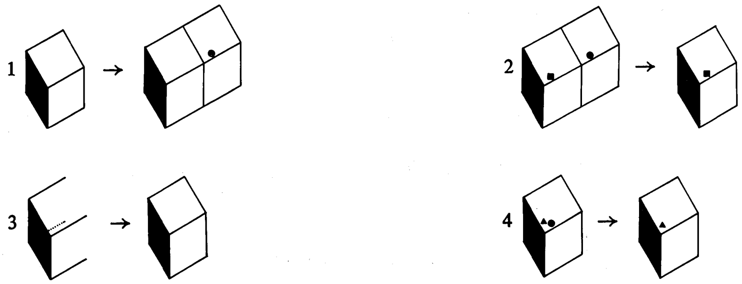


Figure 36. Anomalies resulting from the application of a shape rule of type 2.

shape grammars like the one given in figure 37. The language defined by this grammar contains all possible designs that can be constructed with cubes of a fixed size by adding and subtracting such cubes according to the spatial relation for gift 3. No other designs are in the language.

With the exception of the fifth and seventh ones, all of the shape rule schemata in the grammar are given by shape rules. The first and second ones are of types 3 and 4, respectively. The first adds a cube with the symbol • at its centroid to a design; the second subtracts a cube with the symbol ■ at its centroid from a design whenever this labelled cube and a cube with the symbol ■ at its centroid have the required spatial relation. The third shape rule provides for the replacement of a face of a cube that has been erased inadvertently by an application of the second shape rule. The fourth



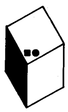
5 $\langle s_\phi, \{(0, 0, 0){:}\blacksquare\}\rangle \rightarrow \langle s_\phi, \{(x, y, z){:}\blacksquare\}\rangle$

6 $\langle s_\phi, \{(0, 0, 0){:}\blacksquare\}\rangle \rightarrow \langle s_\phi, \{(0, 0, 0){:}\blacktriangle\}\rangle$

7 $\langle s_\phi, \{(0, 0, 0){:}\blacktriangle\}\rangle \rightarrow \langle s_\phi, \{(x, y, z){:}\blacktriangle\}\rangle$

8 $\langle s_\phi, \{(0, 0, 0){:}\blacktriangle\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rule schemata



initial shape

Figure 37. A parametric shape grammar in which parts of designs inadvertently erased by applying a shape rule of type 4 can be replaced.

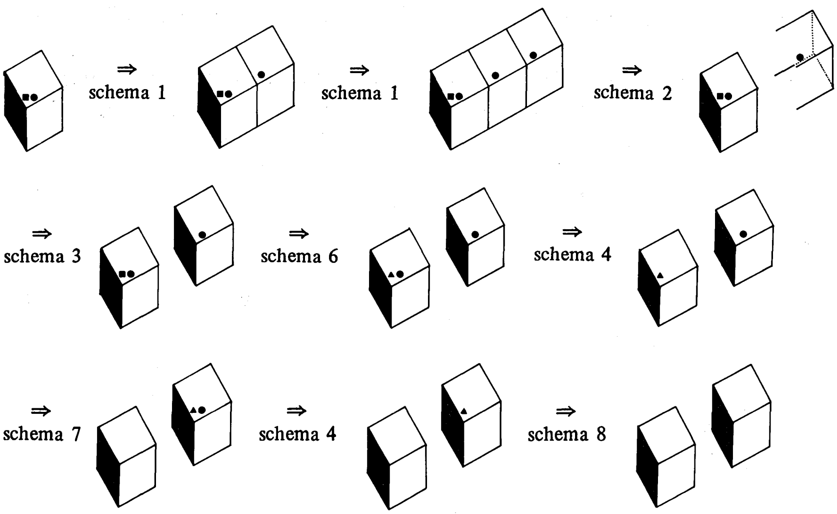


Figure 38. Constructing a design with the parametric shape grammar of figure 37.

shape rule erases the symbol ● at the centroid of a complete cube whenever the symbol ▲ is also associated with this point. The fifth and seventh shape rule schemata allow for the symbols ■ and ▲, respectively, to be moved from the centroid of one cube to the centroid of any other. The sixth shape rule replaces the occurrence of the symbol ■ in a design with the symbol ▲. The eighth shape rule erases the symbol ▲.

The initial shape of the grammar consists of a cube with both of the symbols ● and ■ associated with its centroid.

The use of the grammar to construct the completed version of the design of figure 36 is shown in figure 38. All shape rules, whether given explicitly or defined by schemata, are applied under isometries. Essentially, a cube can be subtracted from a design only when it is adjacent to another cube with the symbol ■ at its centroid. After a subtraction, the third shape rule should be applied to replace the faces of cubes inadvertently erased. Once the symbol ■ has been changed to the symbol ▲, no further cubes can be subtracted from a design. The fourth shape rule in conjunction with the seventh schema is then applied to check that all cubes remaining in the design are complete. All of these cubes and only these cubes are indicated by the symbol ● at their centroids, and thus they may be recognized even when their faces have been erased by subtractions. If a cube is complete, the symbol ● is erased. Otherwise, the symbol ● remains in the design, and disqualifies it as a member of the language defined by the grammar.

In the following sections, the shape rules and schemata required to ensure that shape rules of types 2 and 4 apply properly to perform a subtraction are not given explicitly in shape grammars; they are assumed. Shape rules of types 2 and 4 are thus considered to apply without unpleasantness in the construction of designs.

Initial shapes

In a shape grammar, the construction of designs begins with a fixed initial shape to which shape rules based on a given vocabulary of shapes and spatial relations between them are applied. For these shape rules to apply to the initial shape, it must be made up of shapes in the vocabulary and have one or more subshapes that are similar to the left-hand sides of shape rules. When the shape rules are of types 3 and 4, initial shapes are usually labelled.

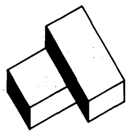
It may sometimes be desirable to define languages of designs constructed from multiple initial shapes by applying the same shape rules. In this case, a 'dummy' initial shape can be specified to consist of, for example, a single labelled point. Shape rules with this new initial shape as their left-hand sides and one of the original initial shapes as their right-hand sides can then be defined. Each application of one of these shape rules results in one of the desired initial shapes.

Shape grammars

Shape grammars incorporating some of the ideas developed above are used in this section to define languages of designs. These shape grammars are all based on the oblong, pillar, and square of gift 6 and spatial relations between these volumes. Every attempt has been made to illustrate particular techniques for defining shape grammars with ones that are as simple as possible. As a result, the designs constructed by these shape grammars often have a conspicuous structure. Still, these designs have strong visual appeal and show considerable visual variety. Of course, more subtle or elaborate use of these techniques can be imagined easily.

In the following figures, shape grammars are presented in a standard format: each one is defined by giving its shape rules and initial shape; a representative design in the language it defines is then shown. Whenever possible, shape rules are drawn to correspond with the drawings of appropriate spatial relations for gift 6. In order to show the intended application of shape rules or the location of labels in them to best advantage, however, shape rules may sometimes be drawn to correspond with other views of these spatial relations. Designs are drawn from the same view as the initial shapes from which they are constructed.

Eight shape grammars defined in terms of the single spatial relation between two oblongs shown in figure 39 are given in figure 40. Each of these shape grammars



6-2

Figure 39. A spatial relation between two oblongs.

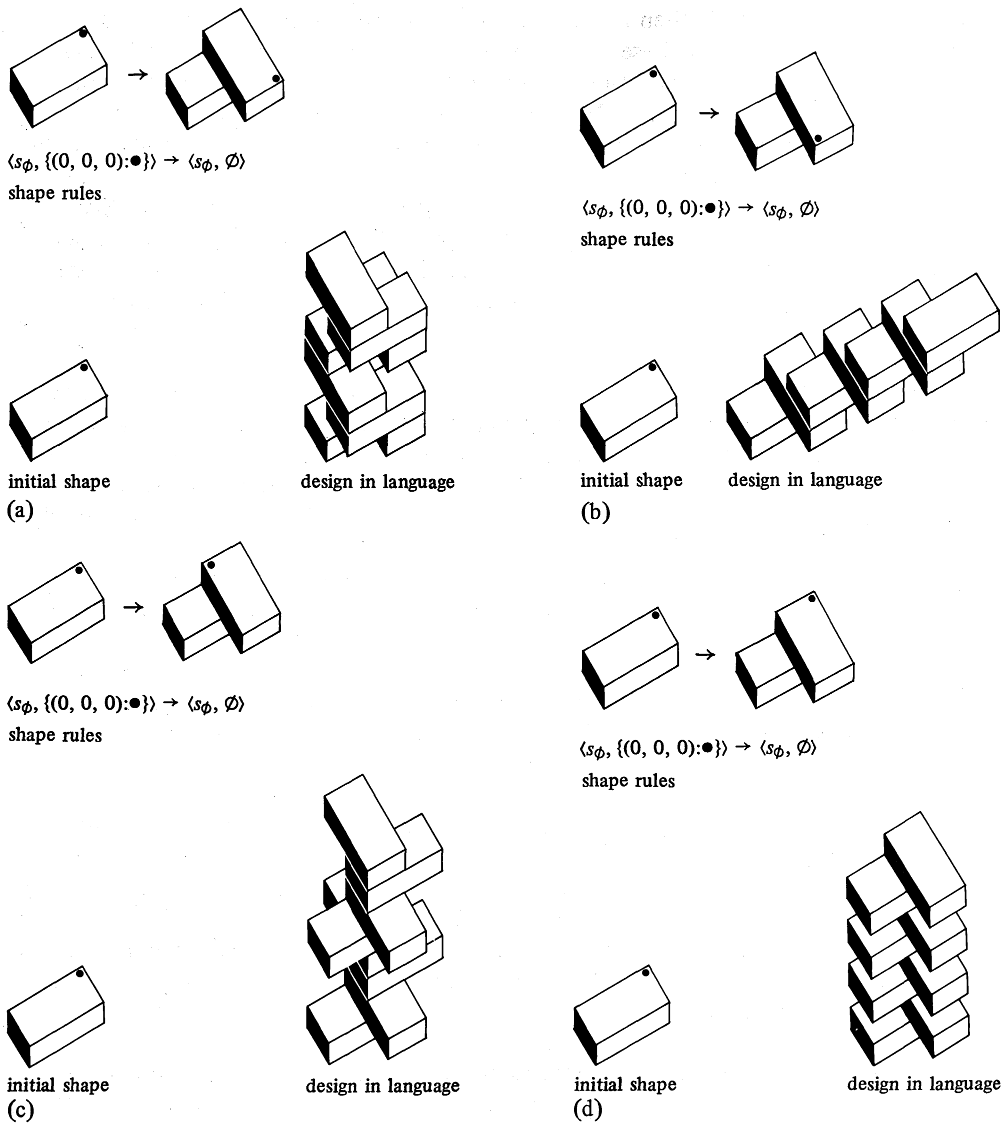
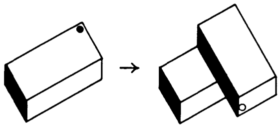


Figure 40. Eight shape grammars based on the spatial relation of figure 39. In these shape grammars, one label is associated with the new oblong in the right-hand sides of shape rules of type 3.

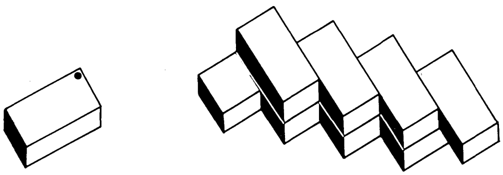
contains one shape rule of type 3 which adds an oblong to a design. This oblong and the one previously added to the design always have the required spatial relation of figure 39. The oblong in the left-hand side of this shape rule is labelled according to its symmetry group as indicated in figure 26; the labelled oblong added in the right-hand side corresponds to one of the eight distinct transformations of the left-hand side determined by the elements of the symmetry group of the oblong. The shape grammars of figure 40 are thus a complete enumeration of shape grammars containing one shape rule of type 3 defined in this way. The initial shape in each of these shape grammars is a labelled oblong as in the left-hand sides of shape rules of type 3. The languages defined by the shape grammars of figure 40 are all distinct, even though they are all based on the same vocabulary (an oblong) and the same spatial relation. The symmetry properties of designs in individual languages are different because of the way oblongs in shape rules of type 3 are labelled. Notice that the languages defined by the shape grammars of figures 40(a)–(e) are potentially infinite, whereas those defined by the shape grammars of figures 40(f)–(h) are finite.

Shape grammars may also contain combinations of the shape rules of type 3 used singly in the shape grammars of figure 40. In these shape grammars, an oblong can be added to the most recently added oblong in a design, but not the initial one, according to the spatial relation of figure 39 in multiple ways corresponding to



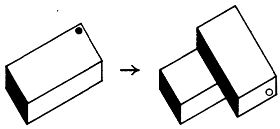
$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules



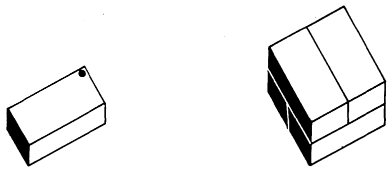
initial shape design in language

(e)



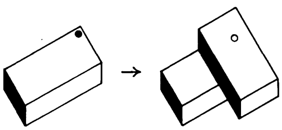
$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules



initial shape design in language

(f)

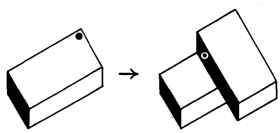


$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules

initial shape design in language

(g)



$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules

initial shape design in language

(h)

Figure 40 (continued)

particular elements in the symmetry group of the oblong. Two such shape grammars are given in figure 41. In the first, two shape rules of type 3 are employed. An oblong can thus be added to a design in two different ways. In the second, three shape rules of type 3 are employed. In this case, an oblong can be added to a design in three different ways. Because the shape grammars of figure 41 include multiple shape rules of type 3 that can be applied disjunctively, they define languages containing designs with variable symmetry properties. In contrast, notice that the shape grammars of figure 40 define languages containing designs with fixed symmetry properties.

Still other shape grammars can be defined in terms of the spatial relation between the two oblongs of figure 39. One simple possibility is to allow either or both of the oblongs in the right-hand side of a shape rule of type 3 to be labelled to correspond to multiple transformations of its left-hand side. The symmetry group of the oblong has order eight. Thus there are $2^8 \cdot 2^8 = 2^{16}$ different shape rules of this type. These shape rules may be used singly or multiply in shape grammars. Two shape grammars in which multiple labels are associated with the new oblongs in the right-hand sides of shape rules of type 3 are given in figures 42(a) and (b); two shape grammars in which labels are associated with both oblongs in the right-hand sides of shape rules of type 3 are given in figures 42(c) and (d). Notice that the shape grammar containing the shape rule of type 3 based on the spatial relation of figure 39 with all sixteen labels in its right-hand side defines the same language of designs as

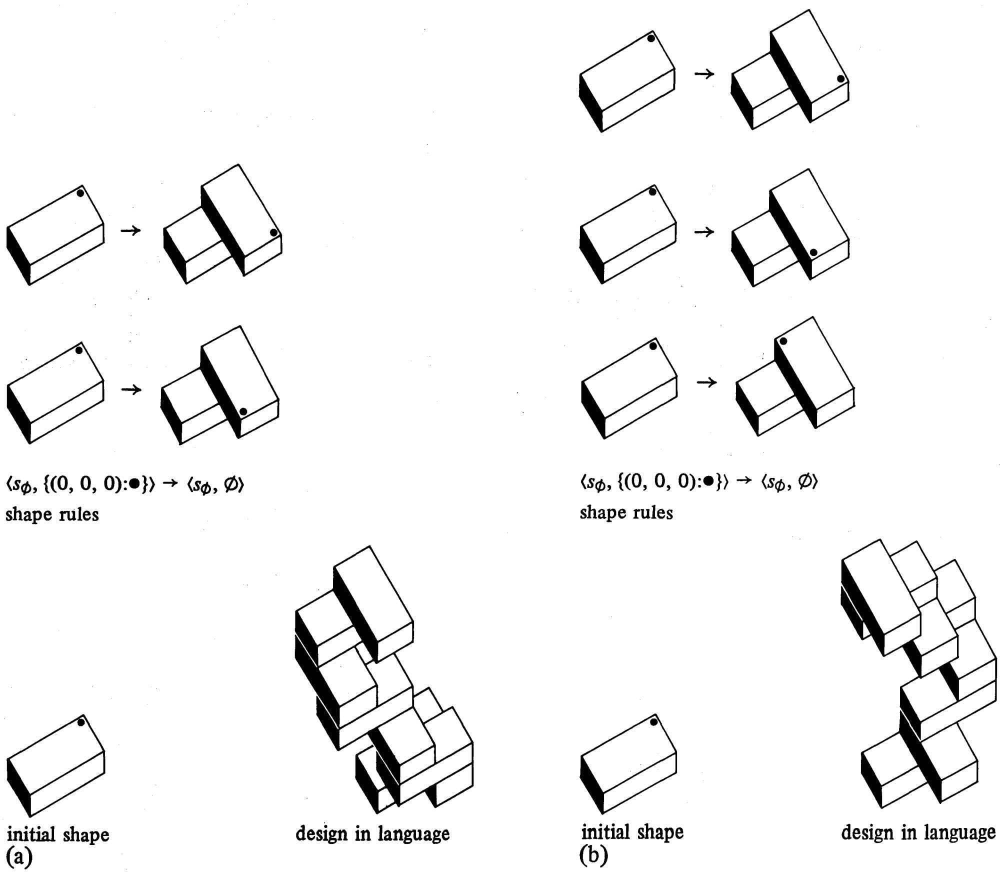


Figure 41. Shape grammars with multiple shape rules of the kind used in figure 40.

the shape grammar containing a single shape rule of type 1 based on this spatial relation.

Distinct symbols can be used to label the oblongs in the shape rules of type 3 contained in the shape grammars of figures 40–42. In this way, applications of shape rules can be distinguished to construct differentiated parts of designs. Four shape grammars in which two distinct symbols are used to label the oblongs in shape rules of type 3 based on the spatial relation of figure 39 are specified in figure 43. In the first two of these shape grammars, applications of shape rules must alternate one after the other; in the last two, applications of shape rules are independent of one another. Notice that the language of designs defined by the shape grammar of figure 43(a) is a subset of the language of designs defined by the shape grammar of figure 41(a).

Shape rules of types 3 and 4 may be based on particular spatial relations and yet be used to construct designs in which none of these spatial relations actually occur.

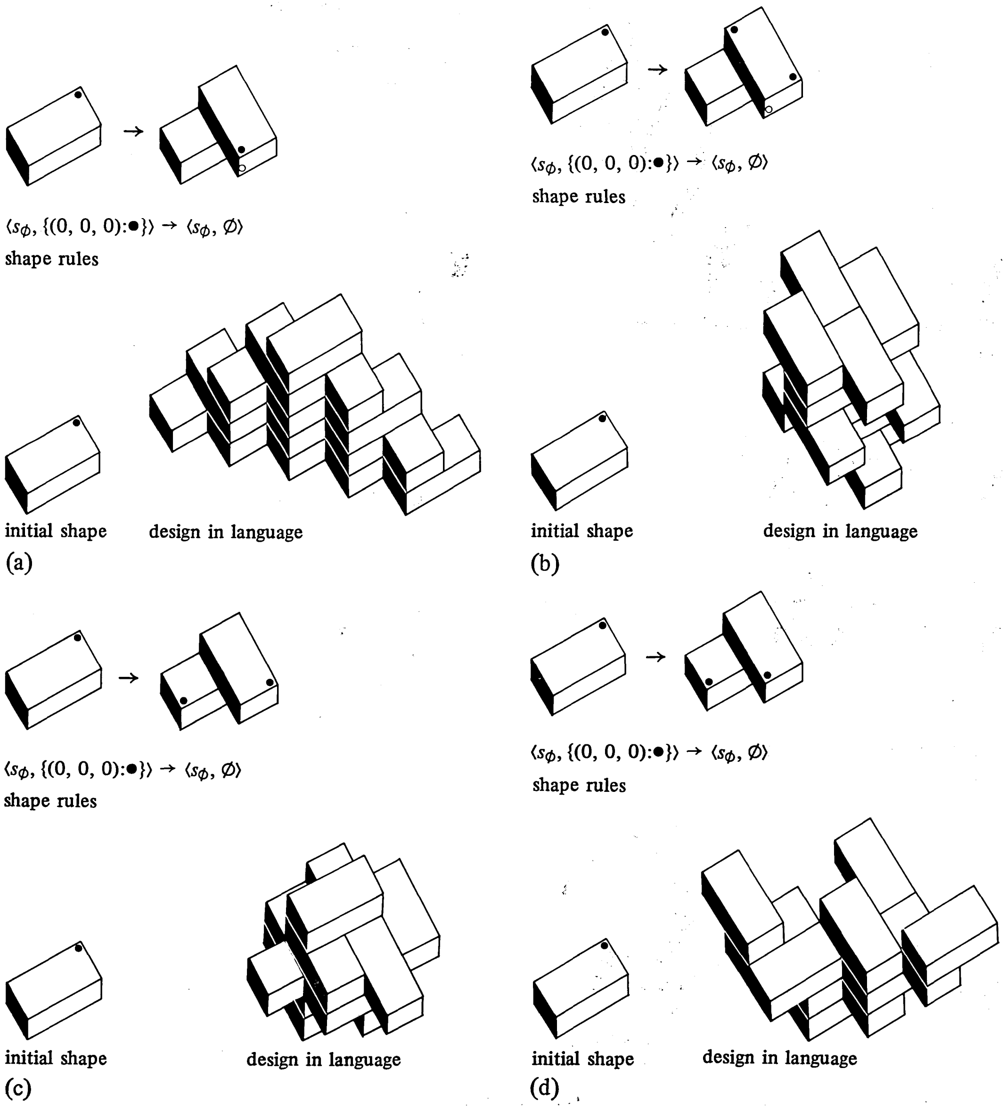


Figure 42. Shape grammars with shape rules having multiple labels in their right-hand sides.

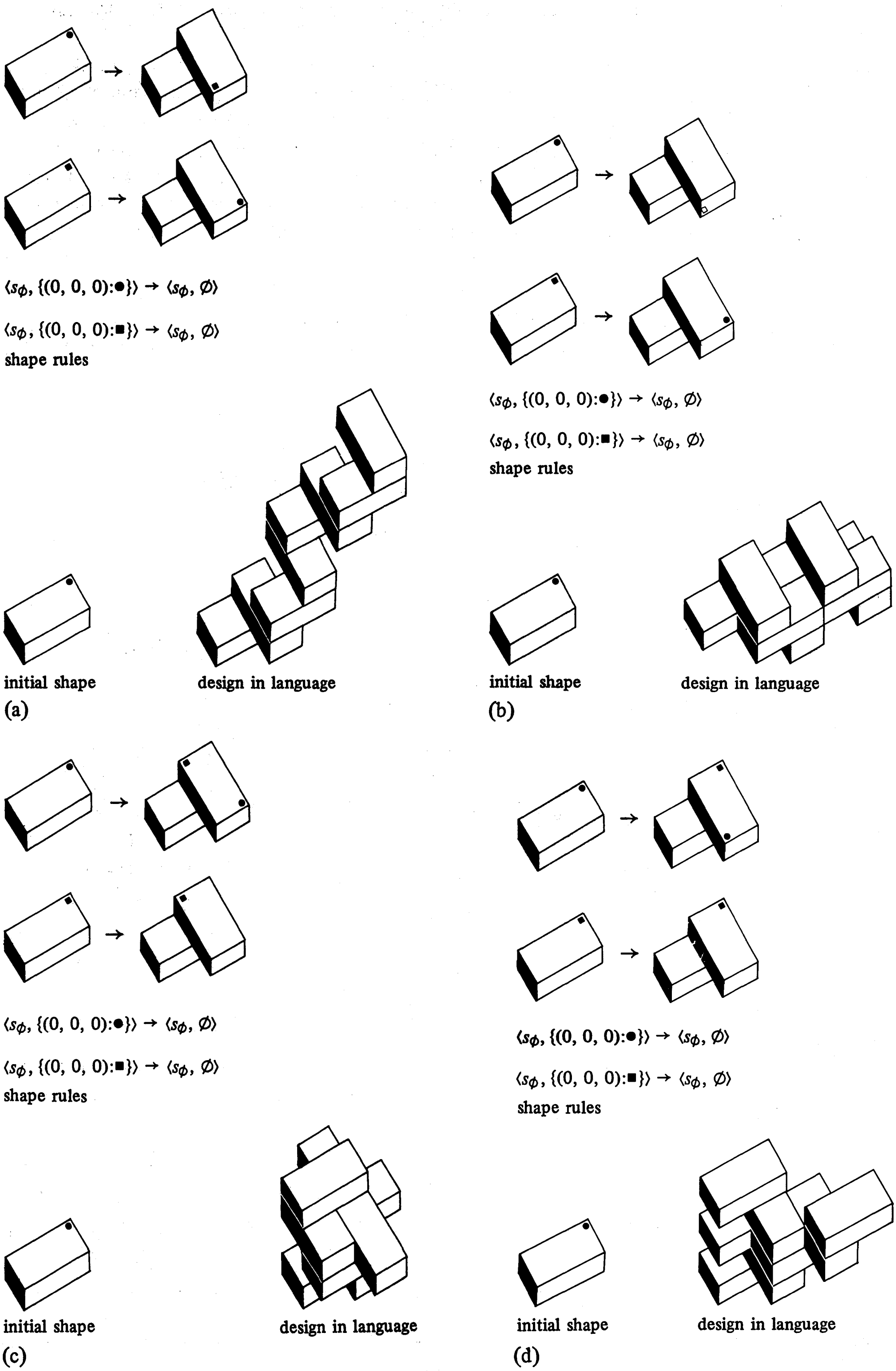


Figure 43. Shape grammars using shape rules with distinct labels.

For example, shape rules of types 3 and 4 based on the spatial relation between two oblongs of figure 39 are contained in the shape grammar of figure 44. No design in the language defined by this shape grammar has an occurrence of this spatial relation. The spatial relations observed in the designs in this language may be considered to be derived from the one given in figure 39.

Languages of designs defined by shape grammars containing shape rules of types 2 and 4 illustrate one difficulty with the *grammatical inference problem*, which may be stated as follows: given a finite corpus of designs in a potentially infinite language, find the simplest shape grammar that defines the language. As the spatial relations used to construct the designs in a language need not occur in them, in general there is no sure way of knowing the actual spatial relations on which the construction of these designs is based. (Of course, things could be worse: the vocabulary of shapes on which designs are based may not even be known.) The solution of the grammatical inference problem usually depends on the identification of the hidden structure underlying designs. In many interesting cases, however, almost all vestiges of this structure may have been erased.

So far, some simple modifications and elaborations of the shape rules in shape grammars have been considered in order to define new languages of designs based on the spatial relation of figure 39. The initial shapes of shape grammars can also be varied to obtain new languages of such designs. In figure 45, for example, the initial shapes in the shape grammars of figures 40(a)–(d) are modified so that they correspond to two distinct transformations of an oblong labelled according to its symmetry group. In this way, new possibilities for design are introduced. Notice that the languages of designs defined by the shape grammars of figures 40(a)–(d) are subsets of the languages of designs defined by the shape grammars of figures 45(a)–(d), respectively. The initial shape employed in the shape grammars of figure 46 is obtained from the initial shape employed in the shape grammars of figure 45 by distinguishing the

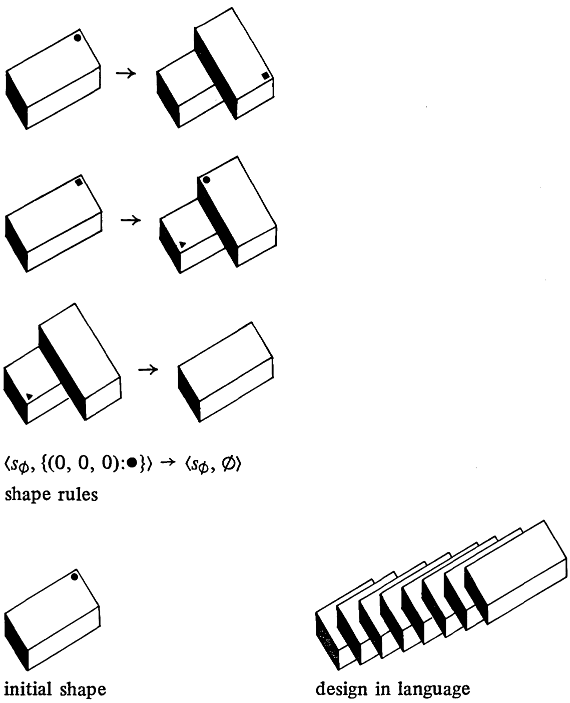


Figure 44. A shape grammar using shape rules of types 3 and 4.

symbols used to label it. Different parts of this new initial shape can thus lead to differentiated parts of designs. An initial shape can consist of multiple oblongs arranged in a particular way. The shape rules of figure 47(a) define one language of designs when used with the initial shape of figure 47(b) consisting of two distinctly labelled oblongs, and still another language of designs when this initial shape is changed by reversing its labels as shown in figure 47(c).

Of course, different spatial relations between two oblongs can be used to obtain shape grammars of the kinds considered above. For example, the spatial relation of figure 48 is used in the four simple shape grammars of figure 49, which vary in the same way as those in figure 40. The visual effects produced by designs often depend on the spatial relations used to construct them.

Shape grammars may also be defined in terms of a single spatial relation between two nonsimilar shapes. For example, four shape grammars based on the spatial relation between an oblong and a pillar of figure 50 are given in figure 51. Each of these shape grammars contains two shape rules of type 3. The first one adds a pillar to a design so that it has the required spatial relation with the most recently added oblong in the design. The oblong in the left-hand side of this shape rule is labelled

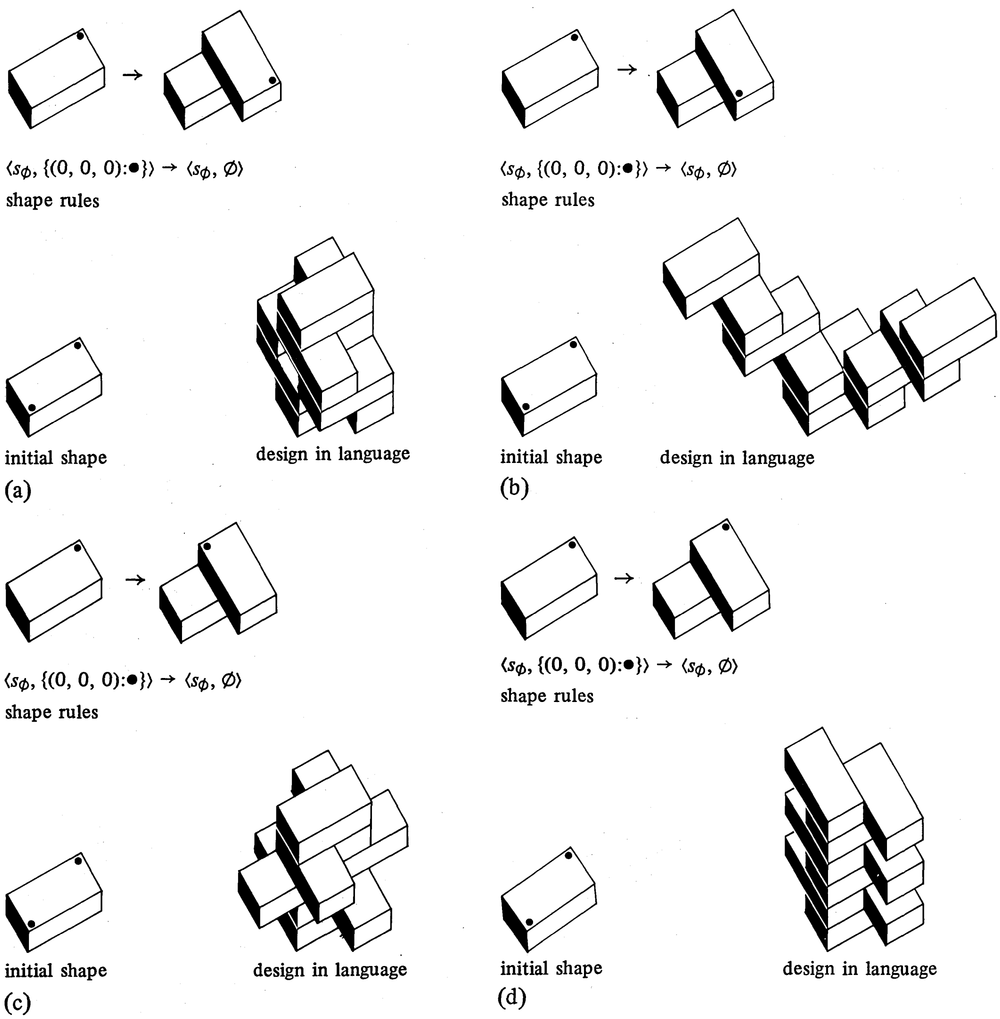
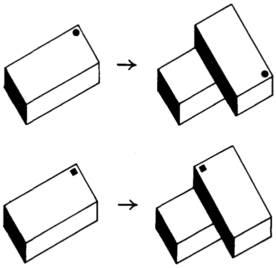


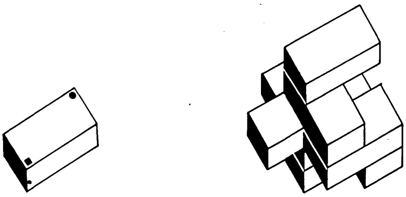
Figure 45. Shape grammars with multiple labels associated with their initial shapes.



$\langle s_\phi, \{(0, 0, 0):\bullet\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

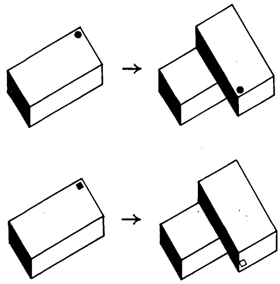
$\langle s_\phi, \{(0, 0, 0):\blacksquare\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

shape rules



initial shape design in language

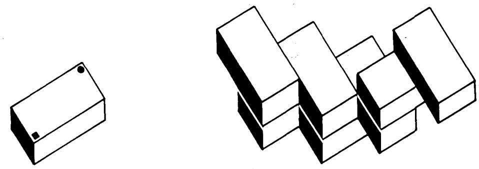
(a)



$\langle s_\phi, \{(0, 0, 0):\bullet\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

$\langle s_\phi, \{(0, 0, 0):\blacksquare\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

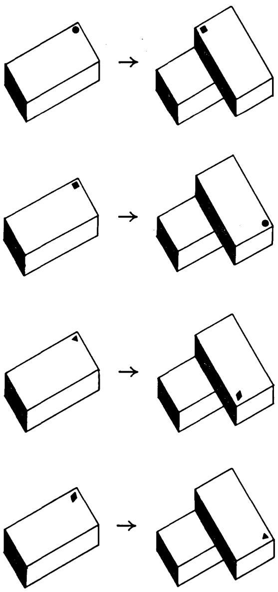
shape rules



initial shape design in language

(b)

Figure 46. Shape grammars with distinct labels associated with their initial shapes.



$\langle s_\phi, \{(0, 0, 0):\bullet\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

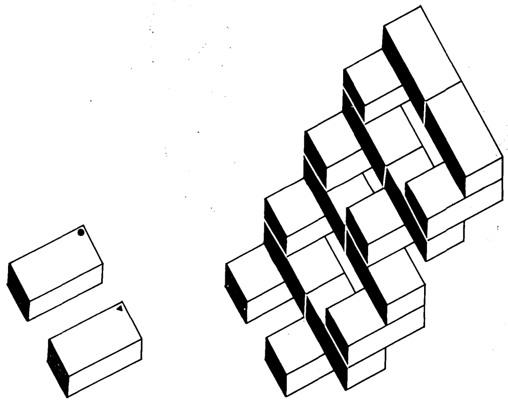
$\langle s_\phi, \{(0, 0, 0):\blacksquare\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

$\langle s_\phi, \{(0, 0, 0):\blacktriangle\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

$\langle s_\phi, \{(0, 0, 0):\blacklozenge\}\rangle \rightarrow \langle s_\phi, \varnothing\rangle$

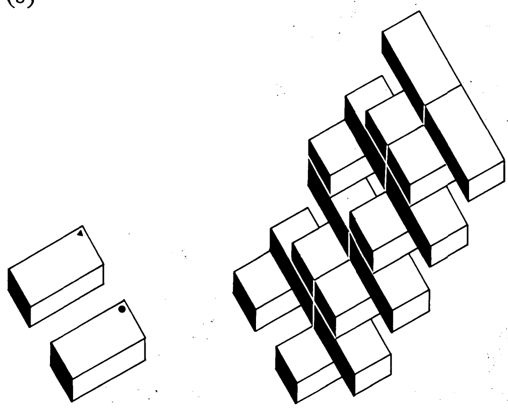
shape rules

(a)



initial shape design in language

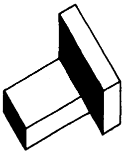
(b)



initial shape design in language

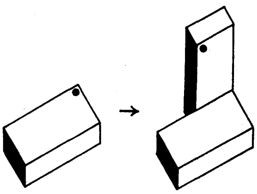
(c)

Figure 47. Shape grammars with initial shapes consisting of multiple oblongs.



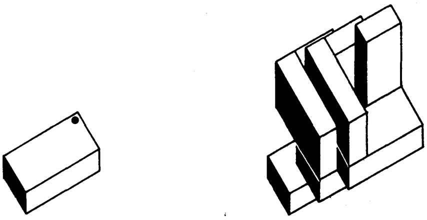
6-6

Figure 48. Another spatial relation between two oblongs.



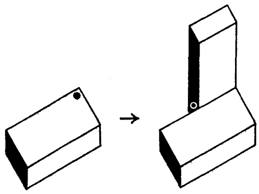
$\langle s_\phi, \{(0, 0, 0)\text{:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules



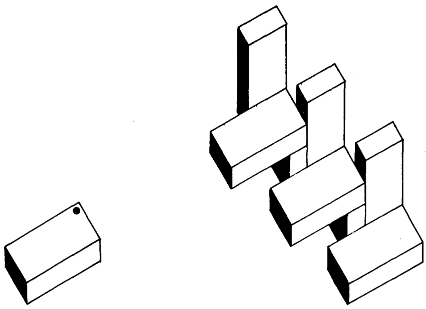
initial shape design in language

(a)



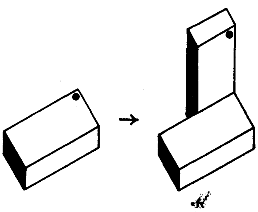
$\langle s_\phi, \{(0, 0, 0)\text{:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules



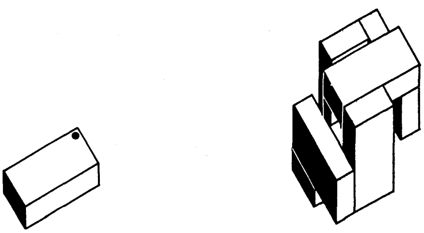
initial shape design in language

(b)



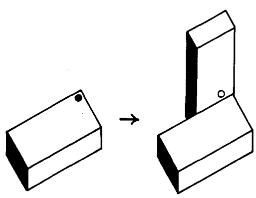
$\langle s_\phi, \{(0, 0, 0)\text{:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules



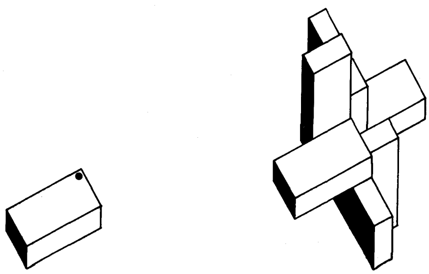
initial shape design in language

(c)



$\langle s_\phi, \{(0, 0, 0)\text{:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

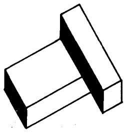
shape rules



initial shape design in language

(d)

Figure 49. Shape grammars of the kind illustrated in figure 40, but based on the spatial relation of figure 48.



6-14

Figure 50. A spatial relation between an oblong and a pillar.

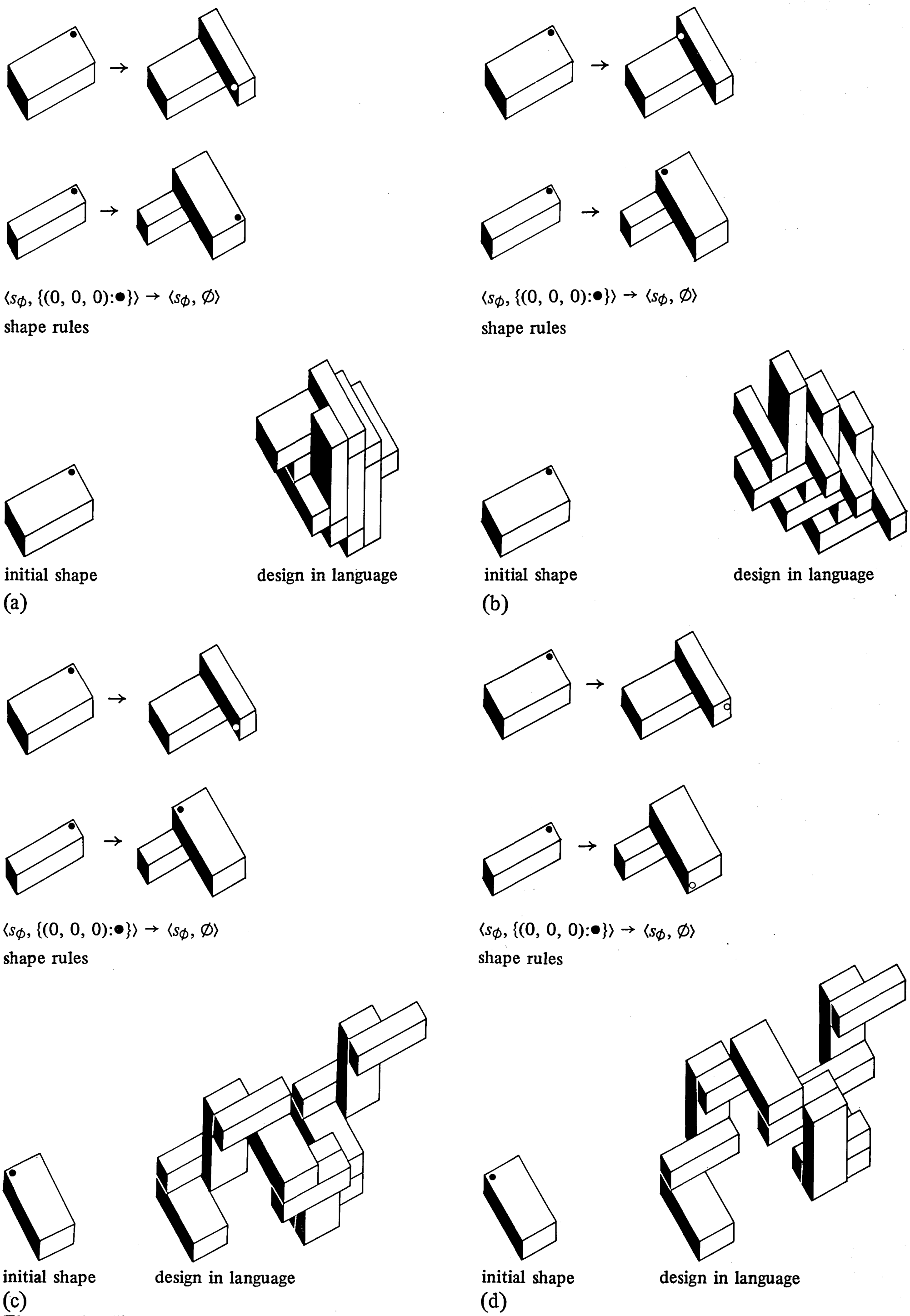


Figure 51. Shape grammars with shape rules of type 3 based on the spatial relation of figure 50.

according to its symmetry group; the labelled pillar in the right-hand side corresponds to a distinct transformation of the left-hand side of the second shape rule. This shape rule complements the first one by adding an oblong to a design so that it has the required spatial relation with the most recently added pillar in the design. The pillar in the left-hand side of the second shape rule is labelled according to its symmetry group (see figure 26); the labelled oblong in the right-hand side corresponds to a distinct transformation of the left-hand side of the first shape rule. The symmetry groups of an oblong and a pillar have orders eight and sixteen, respectively. Thus there are 8 • 16, or 128, distinct pairs of complementary shape rules of this kind. Each of these pairs provides the basis for a shape grammar like the ones in figure 51. No two languages defined by the shape grammars of figure 51 are the same. Any of these shape grammars can be extended or modified as discussed above to define new shape grammars.

Multiple spatial relations between similar and nonsimilar shapes are often employed to construct designs by adding and subtracting shapes. Several different combinations of the spatial relations given in figure 52 are used in the shape grammars of figures 53 and 54.

The shape grammars of figure 53 are elaborations of the ones given in figures 40(a) and (d) and figure 49(b). The shape grammar of figure 53(a) is defined for the spatial relations of figures 52(a) and (e). Here, a square is associated with each oblong, but not the initial one, in designs constructed by applying the shape rule of type 3 in the shape grammar of figure 40(a). Designs constructed using the shape grammar of figure 53(b) are based on the spatial relations of figures 52(a), (d), and (g); they are obtained from ones constructed by applying the shape rule of type 3 in the shape grammar of figure 40(d). With the exception of the initial one, a pillar is first associated with each oblong in such designs. A square is then added to these new designs for each pillar already in them. In the shape grammar of figure 53(c), the spatial relations of figures 52(b), (d), and (g) are used. A pillar and a square are associated with each oblong, including the initial one but not the final one, in designs constructed by applying the shape rule of type 3 in the shape grammar of figure 49(b). In general, complicated shape grammars can often be defined by augmenting the shape rules in simpler shape grammars. In this way, the construction of designs involving multiple spatial relations may be thought of in terms of adding shapes to designs based on a single spatial relation. These elementary designs thus provide the underlying structure for the construction of more elaborate ones.

Shape rules of types 3 and 4 are used in the shape grammars of figure 54. The shape grammar of figure 54(a) is based on the spatial relations of figures 52(a), (e), and (f); it allows for alternating oblongs to be removed from designs constructed by

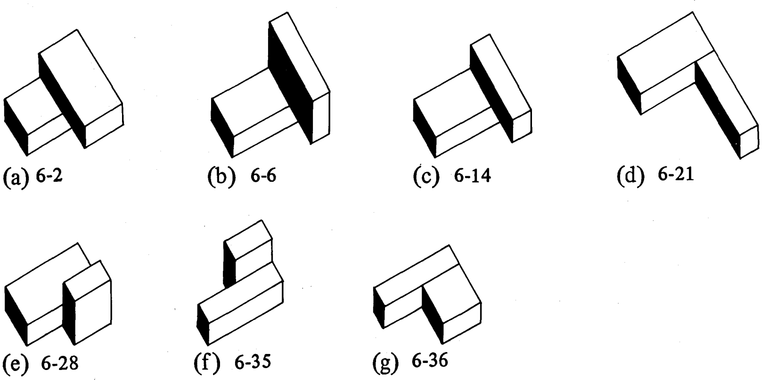
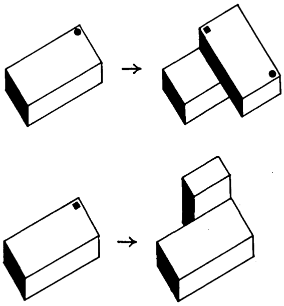
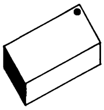


Figure 52. Some spatial relations for gift 6.

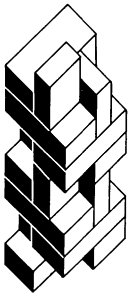


$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules

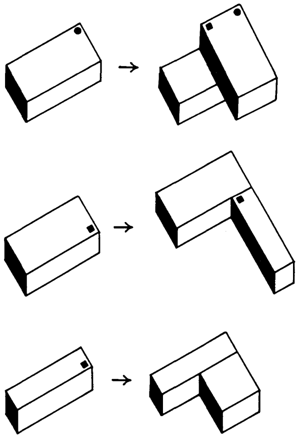


initial shape



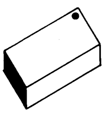
design in language

(a)

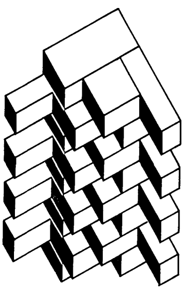


$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules

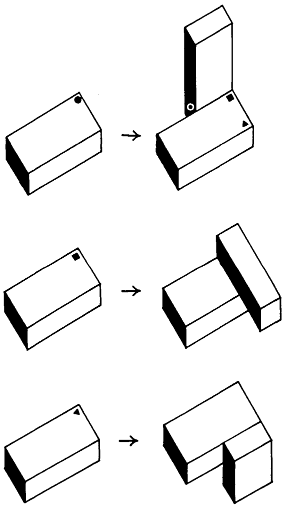


initial shape



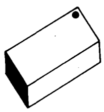
design in language

(b)

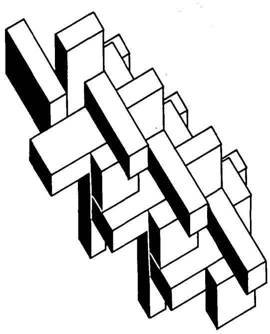


$\langle s_\phi, \{(0, 0, 0){:}\bullet\}\rangle \rightarrow \langle s_\phi, \emptyset\rangle$

shape rules



initial shape



design in language

(c)

Figure 53. Shape grammars with shape rules of type 3 based on multiple spatial relations.

means of the shape rules in the shape grammar of figure 53(a), and then replaced by pillars. Notice that this replacement process requires the use of multiple labels to distinguish those oblongs that are to remain in a completed design from those that are not. Further, all designs so constructed are derived from ones containing an odd number of oblongs. In the shape grammar of figure 54(b), the spatial relations of figures 52(a), (d), and (g) are employed. All of the pillars are removed from designs

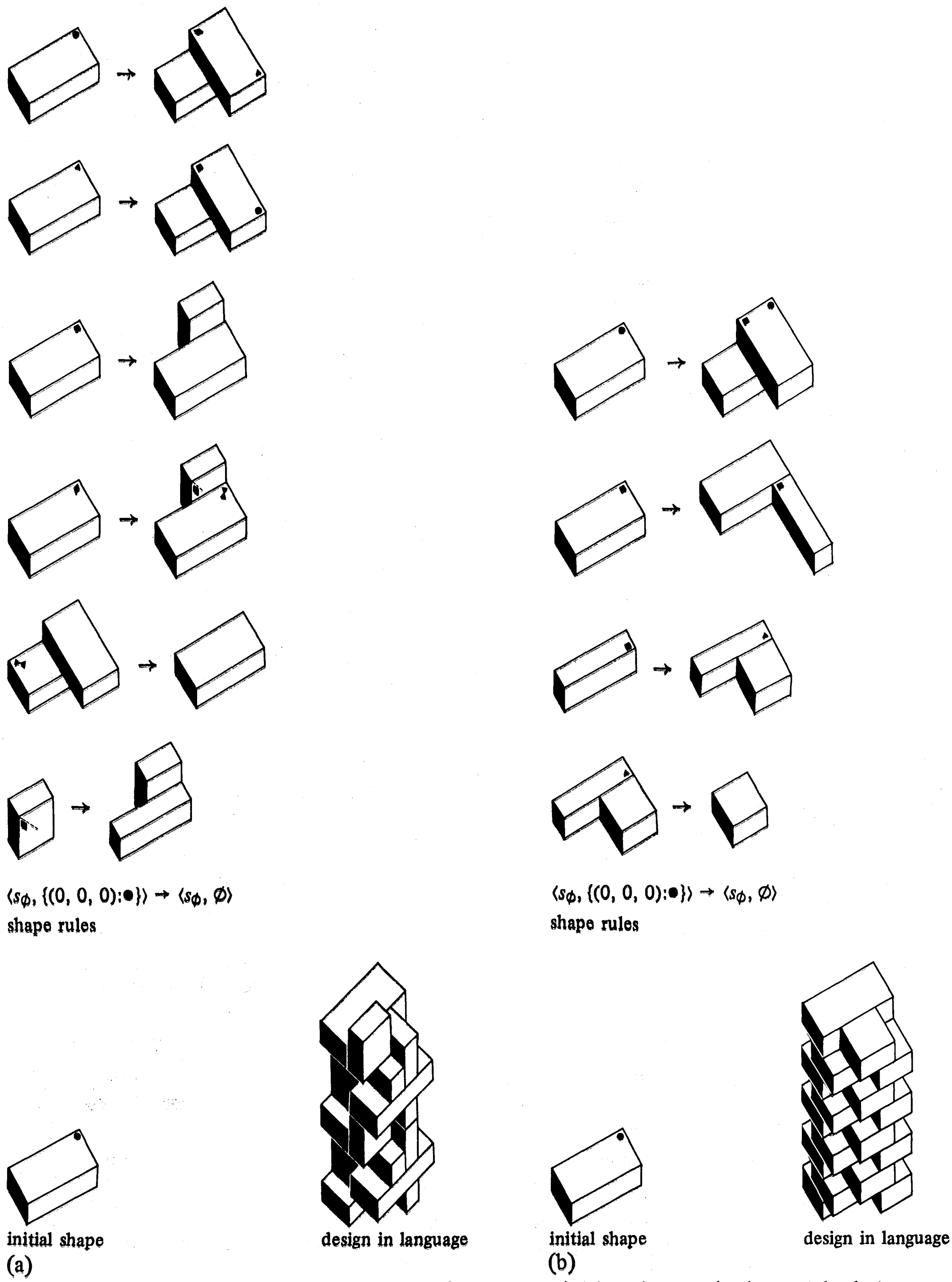


Figure 54. Shape grammars with shape rules of types 3 and 4 based on multiple spatial relations.

constructed by applying the shape rules in the shape grammar of figure 53(b). The shape grammars of figure 54 illustrate again that a simple design can first be constructed, and then used as the underlying structure for the construction of a more complicated one. When shape rules of types 2 and 4 are used, however, this underlying structure may be erased in part or in total.

The examples given in this section merely begin to suggest some of the ways that shape grammars can incorporate the ideas developed in the previous sections. Given a vocabulary of shapes and spatial relations between them, shape grammars can be defined to construct designs made with the shapes in this vocabulary combined according to these spatial relations. These designs may have special properties involving symmetry, spatial ambiguity, interpenetration, and so on.

The techniques described here are not the only ones that can be used to define languages of designs with shape grammars. As shown by Stiny (1975) shape grammars can also be given to construct the designs in languages formed by combining or augmenting other languages of designs in terms of various language-theoretic operations. These operations include, for example, set union, intersection, and difference; shape union, intersection, and difference; homomorphism; and substitution. Simple shape grammars can thus lead to more complicated ones defining very rich languages of designs.

In this paper, shape grammars were applied by hand to construct designs. For shape grammars with more than a few shape rules, however, computer methods of application are usually a more feasible means and, indeed, are often required to construct designs. The shape grammar interpreter described by Krishnamurti (1980; 1981a; 1981b) was developed for this purpose. By means of this computer system, shape grammars can be defined interactively at a computer console, and then applied automatically. It is expected that this computer system will greatly enhance one's ability to define and apply shape grammars, and thus contribute markedly to the constructive understanding of languages of designs and the ability to use them in the studio and practice.

Conclusions

Rules and designs

The programme of figure 5 provides for languages of designs to be defined by moving from a completely unstructured situation where anything is possible (designs constructed by combining transformations of shapes in a given vocabulary by shape union and shape difference) to highly structured ones where only things with special properties are possible (designs constructed by shape grammars based on the vocabulary and spatial relations between the shapes in it). The transition from stage to stage in the programme allows for the definition of *rules* which apply to construct designs.

Thinking about the rules used to construct designs has several advantages:

(1) Rules are usually much less complicated than the designs they produce; they can be framed in terms of simple relationships that correspond to a designer's visual intuitions. These relationships can be enumerated without too much difficulty, thus leading to multiple rules which can be used in various forms and combinations to define different languages of designs. A few simple rules can be used to construct a multiplicity of complicated designs.

(2) Like Froebel's categories, rules open up new avenues or directions for design with a given vocabulary; they increase the designer's power of observation in both the creative and the selective senses described earlier. But where the categories are used in intuitive processes, rules are used in algorithmic ones.

(3) In order to define rules that can be used in algorithms (shape grammars), a designer must represent his ideas and knowledge about possibilities for design in an explicit

and detailed way. Once these ideas and this knowledge have been so represented, their implications for design can be determined by applying the rules to construct designs. In this way, the designer acquires command of a language of designs which is not only accessible to him via the rules defining it but to anyone else who cares to learn these rules.

(4) Rules shift the emphasis in design away from individual designs to languages of designs. By thinking about rules, a designer can concentrate on the constructive basis for his designs and thus develop an explicit awareness of their properties and structure. This understanding allows the designer to use his past experience in new situations; it pertains to rules which can be applied to construct yet new designs and is thereby not restricted to individual designs which are appropriate only in special situations.

(5) When a designer uses a language of designs defined by rules, he can examine a design and its variations without loss of understanding simply by applying the rules to construct them. This opportunity is seldom to be had when individual designs are modified in an ad hoc manner to obtain variations. The consequences of succeeding changes usually become more and more obscure, and thus changes are less and less likely to be made. Without rules to construct alternative designs, the designer often settles for his initial design or one of its early variations, even when better designs are within his grasp.

(6) As envisaged here, languages of designs defined by rules need not lead to the usual combinatorial difficulties of far too many possibilities and far too little knowledge about them. Indeed, enumeration and other counting techniques are often superfluous, as the properties and structure of individual designs can be derived in some detail from the rules used to construct them. And even when enumeration is required, it can be guided effectively and efficiently by these very same rules.

(7) Rules can be modified systematically to define new languages of designs that reflect changing circumstances or incorporate new ideas. In this way, a designer can acquire the use of new languages of designs from the ones he already knows. He is able to experiment with each of these new languages and to determine its possibilities and potential in different situations, not because he is familiar with its individual members, but because he knows the rules to construct them.

Research aimed first at developing new methods to specify rules to define languages of designs and then at more powerful techniques to describe the special properties of designs in such languages in terms of the rules used to define them is thus expected to lead to a more complete and detailed understanding of the constructive bases of design. The ideas presented in this paper are merely a start in this direction. Much more needs to be done, especially if the relationship between the rules used to construct designs and the properties of designs is to be employed reciprocally, both to obtain rules that define languages of designs with *given* properties and to describe the properties of individual designs in languages defined by *given* rules. Once new ways are discovered to handle this kind of relationship, questions about design that now seem too amorphous even for illegible formulation much less rigorous solution should become accessible to systematic thought.

Languages and design

Particular languages of designs are just one component in the *design machines* described originally by Stiny and Gips (1978a), and more recently by Stiny and March (1981). For a language to be used in design, it must first be *interpreted* and *ordered.* As suggested in this paper, a language of designs may be interpreted *constructively* in terms of the rules used to construct designs; alternatively, it may be interpreted *evocatively* in terms of the conventions (rules) by which designs are connected to a complex of associations and ideas; often, it is interpreted both constructively and evocatively. In general, a language of designs is partially ordered, but in many interesting cases, it may be totally ordered (for example, see Stiny and Gips, 1978b). Once a language of designs has been interpreted and ordered, it must then interface with specific kinds of *design contexts* pertaining to specific requirements of purpose and production via an appropriate *theory* about designs and the way individual ones fit such contexts. The description of these design contexts and the application of this theory are both rule based. These rules and the rules by which a language of designs is interpreted and ordered constitute a design machine.

The common structure of all design machines should provide the basis for a future science of design. The elaboration of this science depends on the success with which particular design machines can be specified by giving the rules comprising them. Developing an understanding of languages of designs is a necessary step in this direction. The constructive programme outlined in this paper provides some machinery to obtain from scratch new languages of designs that are interpreted constructively. Other languages of designs have been interpreted constructively by inferring the rules used to construct designs from corpora of existing ones. Notable examples include traditional Chinese lattice designs (Stiny, 1977), Palladian villa plans (Stiny and Mitchell, 1978), Mughul gardens (Stiny and Mitchell, 1980), Hepplewhite chair-back designs (Knight, 1980), Japanese tearooms (Knight, 1981), and the architecture of Guiseppe Terragni (Flemming, 1981). Indeed, the constructive approach to languages of designs, whether defined from scratch or based on existing examples of designs, is now becoming an established research paradigm.

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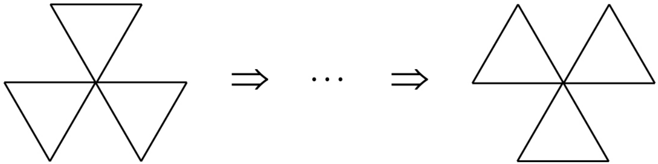

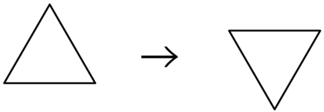

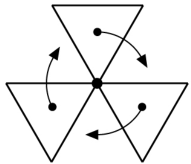

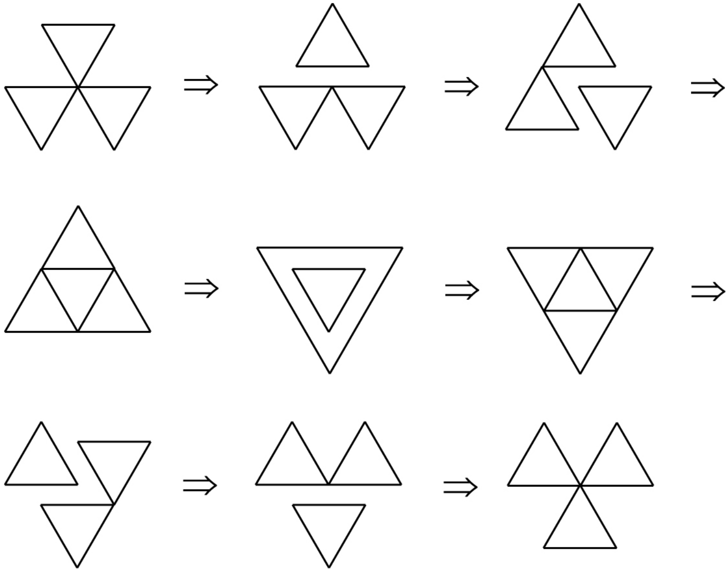

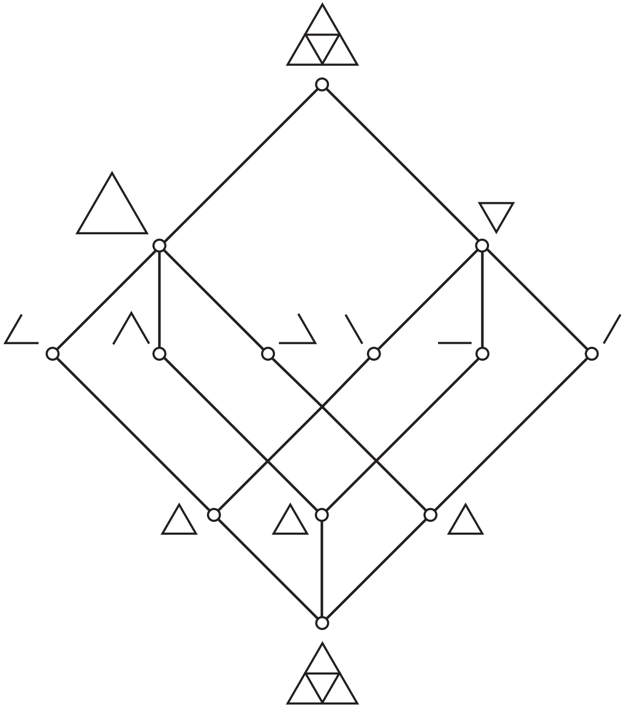

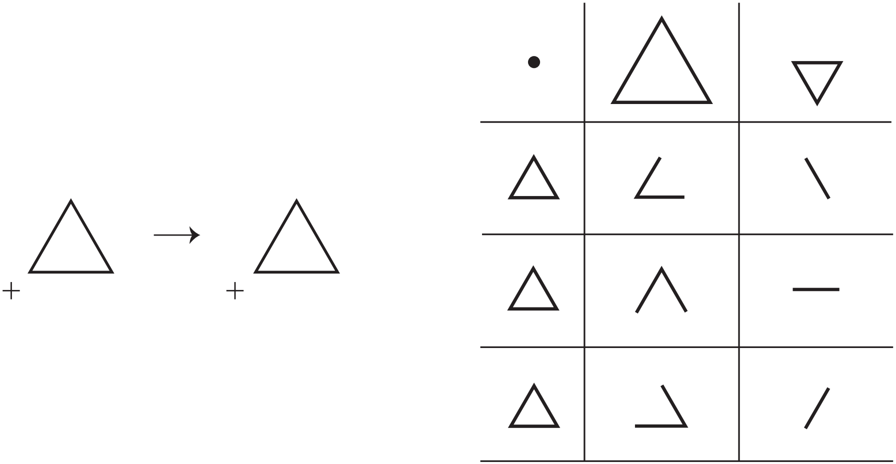

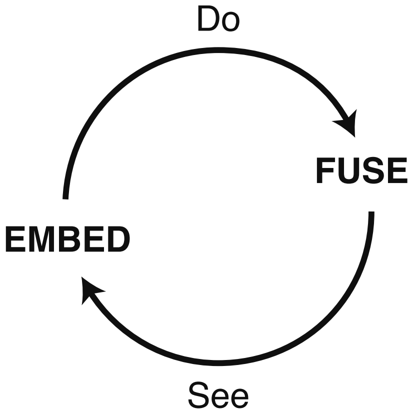
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